

UKMT Senior Mathematical Challenge 2026

Solutions

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Abstract

This document contains my solutions to all 25 problems of the 2026 UKMT Senior Mathematical Challenge. Questions 1 to 22 are multiple choice, while Questions 23 to 25 have a numerical answer between 000 and 999. I have tried to keep each solution concise, leading with key ideas and leaving computations as an exercises for the reader. All diagrams are redrawn in `tkz-euclide`. More of my write-ups can be found on my website, kevinvariant08.github.io.

Question 1

What is the product of two numbers whose sum is 20 and whose difference is 26?

- A 6 B 46 C 12 D -46 E -69

Solution. Let the two numbers be x and y , where $x > y$. Hence $x + y = 20$ and $x - y = 26$, which solves to get $x = 23, y = -3$, so their product is -69 .

Answer: E

Question 2

Alana buys 4 apples and 5 pears for £3.20. At the same prices, 5 apples and 4 pears cost £3.10. What is the total cost of one apple and one pear?

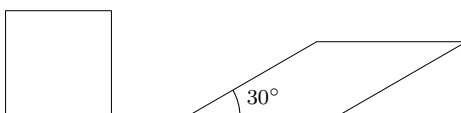
- A 50p B 60p C 70p D 80p E 90p

Solution. Similar to the previous question, let a and p denote the price of an apple and pear in pence, respectively. Hence $4a + 5p = 320$ and $5a + 4p = 310$. Adding the equations yield $9(a + p) = 630$, so $a + p = 70$, which is what we wanted to find.

Answer: C

Question 3

The square and the rhombus with interior angle of 30° have the same area. What is the ratio of the side-length of the square to the side-length of the rhombus?



- A** 1 : 1 **B** 1 : $\sqrt{2}$ **C** $\sqrt{3} : 2$ **D** 2 : 3 **E** 1 : 2

Solution. Let the rhombus have side length 1, so by considering two congruent isosceles triangles, we get the total area $2 \cdot \frac{1}{2} \cdot 1^2 \sin 30 = \frac{1}{2}$, meaning the square must have side length $\frac{1}{\sqrt{2}}$, so the ratio is $\frac{1}{\sqrt{2}} : 1 = 1 : \sqrt{2}$.

Answer: **B**

Question 4

$2^x = \sqrt{4\sqrt{4\sqrt{4}}}$. What is the value of x ?

- A** 3 **B** $\frac{5}{2}$ **C** 2 **D** $\frac{7}{4}$ **E** $\frac{3}{2}$

Solution.

Consider the powers of 2 for the nested surd, keeping in mind $2^2 = 4$. Hence, $x = \frac{\frac{\frac{2}{2}+2}{2}+2}{2} = \frac{7}{4}$.

Answer: **D**

Question 5

A sequence of numbers has the property that for every positive integer n , the mean of the first n terms of the sequence is $\frac{1}{n}$. What is the 1000th term of this sequence?

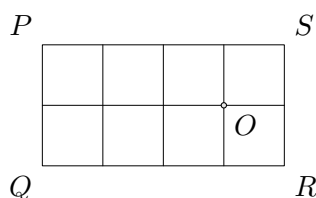
- A** 0 **B** $\frac{1}{999}$ **C** $\frac{1}{1000}$ **D** $\frac{1}{1001}$ **E** $\frac{1}{1000000}$

Solution. From the question, we infer that the sum of all n terms is 1. Listing out the first few terms, we get 1, 0, 0, 0, ..., 0, so clearly, all terms except the first are equal to 0.

Answer: **A**

Question 6

Rectangle $PQRS$ is made from eight 1 by 1 squares as shown in the diagram. At point O , four of those squares meet. The rectangle is rotated through 180° about point O . Point P travels k times as far as point R during that rotation. What is the value of k ?



- A** $\sqrt{2}$ **B** $\sqrt{3}$ **C** 2 **D** $\sqrt{5}$ **E** $\sqrt{6}$

Solution. Every point of the rectangle travels along a semicircle centred at O , so the distance travelled is proportional to the distance from O . Placing Q at the origin, we have $O = (3, 1)$, $P = (0, 2)$ and $R = (4, 0)$. Hence $OP = \sqrt{3^2 + 1^2} = \sqrt{10}$ and $OR = \sqrt{1^2 + 1^2} = \sqrt{2}$, so $k = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$.

Answer: **D**

Question 7

A bag contains 50 yellow and 25 purple beads which are to be arranged into five groups of 15. What is the largest possible number of the groups having more purple than yellow beads?

- A 0 B 1 C 2 D 3 E 4

Solution. For a group of 15 to have more purple than yellow beads, it needs at least 8 purple beads. Since $4 \cdot 8 = 32 > 25$, at most 3 groups can do this. This is achievable: take three groups of 8 purple and 7 yellow, using 24 purple and 21 yellow beads, then split the remaining 1 purple and 29 yellow beads into the last two groups. Hence the answer is 3.

Answer: D

Question 8

The number N is the largest 10-digit integer, all of whose digits are different, which is divisible by 7. What is the remainder when N is divided by 10?

- A 0 B 1 C 2 D 3 E 4

Solution. To make N as large as possible, we keep the leading digits 98765432, so the two candidates are 9876543210 and 9876543201. A quick division gives $9876543210 \equiv 2 \pmod{7}$, so it fails, but then $9876543201 = 9876543210 - 9 \equiv 2 - 9 \equiv 0 \pmod{7}$. Hence $N = 9876543201$, which leaves remainder 1.

Answer: B

Question 9

A string of letters is created by applying the following rule in each iteration: any 'a' is replaced by 'ab', and any 'b' is replaced by letter 'a'. Alison writes down letter 'a' in line 1, then she applies the rule and writes down 'ab' in line 2, and 'aba' in line 3. How many 'a's will appear in line 10 if Alison keeps applying the same rule correctly?

- A 34 B 55 C 64 D 89 E 128

Solution. Let a_n and b_n be the number of a's and b's in line n . Every a produces one a and one b, and every b produces one a, so $a_{n+1} = a_n + b_n$ and $b_{n+1} = a_n$. Hence $a_{n+1} = a_n + a_{n-1}$, which is just the Fibonacci recurrence with $a_1 = a_2 = 1$. Listing out, we get 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, so line 10 has 55 a's.

Answer: B

Question 10

How many seven-digit numbers are there such that each digit is either 4, 5 or 6 and each pair of adjacent digits differ by 1?

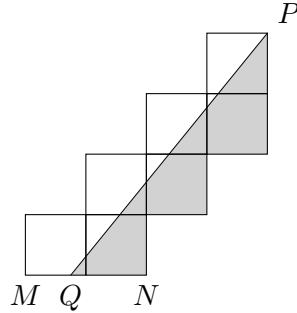
- A 12 B 18 C 24 D 30 E 32

Solution. Since adjacent digits differ by 1, every other digit must be a 5, and the digits in between are each 4 or 6. If the 5's are in positions 1, 3, 5, 7, the other three digits give $2^3 = 8$ numbers. If the 5's are in positions 2, 4, 6, the other four digits give $2^4 = 16$ numbers. In total, there are $8 + 16 = 24$.

Answer: C

Question 11

Seven squares of unit length are aligned edge-to-edge as shown. A straight line is drawn from P to Q , where Q is the point on the line MN such that the shaded area formed is half the total area of the seven squares. What is the ratio $MQ : QN$?



- A 1 : 2 B 2 : 3 C 3 : 5 D 5 : 8 E 8 : 13

Solution. Set M as the origin, so $N = (2, 0)$, $P = (4, 4)$ and $Q = (q, 0)$. The triangle bounded by PQ , the line MN and the vertical through P has area $\frac{1}{2} \cdot (4 - q) \cdot 4 = 2(4 - q)$. However, the 1×1 square below the staircase for $2 \leq x \leq 3$ and the 1×2 rectangle below it for $3 \leq x \leq 4$ are not part of the shaded region (clearly PQ passes above both), so the shaded area is $2(4 - q) - 3 = 5 - 2q$. Setting this equal to $\frac{7}{2}$ gives $q = \frac{3}{4}$, so $MQ : QN = \frac{3}{4} : \frac{5}{4} = 3 : 5$.

Answer: C

Question 12

A cube has a dot at each vertex, at the midpoint of each edge and at the midpoint of each face. How many straight lines can pass through exactly 3 dots?

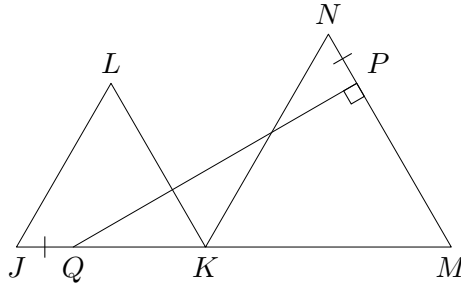
- A 24 B 36 C 48 D 60 E 72

Solution. The dots are exactly the points of a $3 \times 3 \times 3$ grid with the centre of the cube removed, so any line through 3 dots is one of the lines through 3 points of the full grid. There are 27 such lines parallel to an edge, 18 along face diagonals (2 in each of the 9 planes parallel to a face) and 4 space diagonals, giving 49. Now we remove the lines through the centre, which only contain 2 dots: these are 3 edge-parallel lines, 6 face-diagonal lines and the 4 space diagonals, so 13 in total. Hence the answer is $49 - 13 = 36$.

Answer: B

Question 13

JKL and KMN are equilateral triangles with side-lengths 20 and 26 respectively. Points J , K and M lie on a straight line. Points P on NM , and Q on JK , are such that JQ and NP have the same length and QPM is a right angle. The length of PQ is $t\sqrt{3}$. What is the value of t ?



A 24

B 23

C 22

D 21

E 20

Solution. Let $JQ = NP = a$, so $QM = 46 - a$ and $PM = 26 - a$. Since $\angle QMP = 60^\circ$ and $\angle QPM = 90^\circ$, $\triangle QPM$ is a 30-60-90 triangle, so $PM = \frac{1}{2}QM$. Hence $26 - a = \frac{46-a}{2}$, which gives $a = 6$. So $QM = 40$ and $PQ = QM \sin 60^\circ = 20\sqrt{3}$, meaning $t = 20$.

Answer: E

Question 14

Consider the possible expressions formed when each $\square$ is filled with a $+$ or a $-$ sign in

$$\square \frac{1}{1} \square \frac{1}{2} \square \frac{1}{3} \square \frac{1}{4} \square \frac{1}{5} \square \frac{1}{6}.$$

For example, $-\frac{1}{1} + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, which has value $-\frac{43}{60}$, is one of the 2^6 possible expressions. Among these expressions, what is the smallest possible positive value?

A $\frac{1}{60}$

B $\frac{1}{30}$

C $\frac{1}{20}$

D $\frac{1}{10}$

E ?

Solution. Multiplying everything by 60, the terms become 60, 30, 20, 15, 12, 10, which sum to 147. If the terms with a minus sign sum to s , the expression equals $\frac{147-2s}{60}$. Since $147 - 2s$ is odd, the best we could hope for is $\frac{1}{60}$, which needs $s = 73$. As 73 is odd, 15 must be included, and then the rest would need to sum to 58 using 60, 30, 20, 12, 10, which is impossible. The next best is $s = 72$, which works as $60 + 12 = 72$, giving

$$-\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} = \frac{3}{60} = \frac{1}{20}.$$

Answer: C

Question 15

The mean of n numbers is n , $n > 2$. One of the numbers is $4n$ and another is $n - 6$. The other numbers are all equal to k . Which of these expressions gives k in terms of n ?

A $n - 3$

B $n - \frac{5}{2}$

C $n - 2$

D $n - \frac{3}{2}$

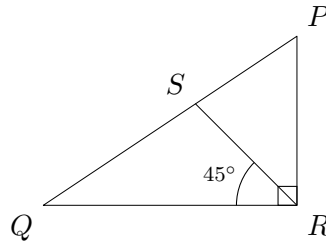
E $n - 1$

Solution. The sum of all n numbers is n^2 , so $4n + (n - 6) + (n - 2)k = n^2$. Hence $(n - 2)k = n^2 - 5n + 6 = (n - 2)(n - 3)$, and since $n > 2$, $k = n - 3$.

Answer: A

Question 16

Triangle PQR has a right angle at R . The lengths of QR and RP are $3\sqrt{5}$ and $2\sqrt{5}$ respectively and $\angle QRS = 45^\circ$. What is the area of triangle PSR ?



A 5

B 6

C $3\sqrt{5}$

D 7.5

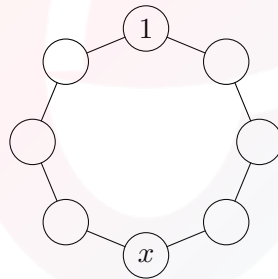
E $7\sqrt{2}$

Solution. First, $[\triangle PQR] = \frac{1}{2} \cdot 3\sqrt{5} \cdot 2\sqrt{5} = 15$. Since $\angle QRS = 45^\circ$, RS bisects the right angle at R , so by the *Angle Bisector Theorem*, $PS : SQ = RP : RQ = 2 : 3$. Triangles PSR and QSR share the same height from R , so $[\triangle PSR] = \frac{2}{5} \cdot 15 = 6$.

Answer: B

Question 17

The numbers 2, 3, 4, 5, 6, 7 and 8 are to be placed in the empty circles so that (i) there is one number in each circle, and (ii) the sum of the numbers in each pair of adjacent circles is prime. How many possible values are there for x ?



A 0

B 1

C 2

D 3

E 4

Solution. Every adjacent sum is at least $1 + 2 = 3$, so it must be an odd prime, meaning odd and even numbers alternate around the ring. As x is opposite 1, it is an even number of steps away, so x is odd, i.e. $x \in \{3, 5, 7\}$. Now 8 can only sit next to 3 or 5 (as $8 + 1 = 9$ and $8 + 7 = 15$), so 3 and 5 are the two neighbours of 8, and hence two steps apart. If $x = 7$, then 3 and 5 fill the remaining two odd circles, which are opposite each other, a contradiction. On the other hand, reading around the ring from 1, the arrangements 1, 2, 5, 8, 3, 4, 7, 6 and 1, 2, 3, 8, 5, 6, 7, 4 give $x = 3$ and $x = 5$ respectively. Hence there are 2 possible values.

Answer: C

Question 18

Let $g(x) = x^{\frac{1}{n}}$. Each of x , a and b are positive real numbers and n is a positive integer. Which of the following is a simplification of

$$g(a) \times g(b^2) \times g(a^3) \times g(b^4) \times \cdots \times g(a^{2n-1}) \times g(b^{2n})?$$

- A** $a^n b^{n+1}$ **B** $a^n b^{n+2}$ **C** $a^{n+1} b^{n+2}$ **D** $a^{2n} b^{2n+2}$ **E** $a^{2n+1} b^{2n+2}$

Solution. Combining the powers, the exponent of a is $\frac{1+3+\dots+(2n-1)}{n} = \frac{n^2}{n} = n$, and the exponent of b is $\frac{2+4+\dots+2n}{n} = \frac{n(n+1)}{n} = n+1$. So the product is $a^n b^{n+1}$.

Answer: A

Question 19

The largest integer less than or equal to x is represented by $\lfloor x \rfloor$. For example $\lfloor 5.8 \rfloor = 5$ and $\lfloor 3 \rfloor = 3$. For how many positive integers x does $\lfloor x^{\frac{1}{2}} \rfloor + \lfloor x^{\frac{1}{3}} \rfloor = 8$?

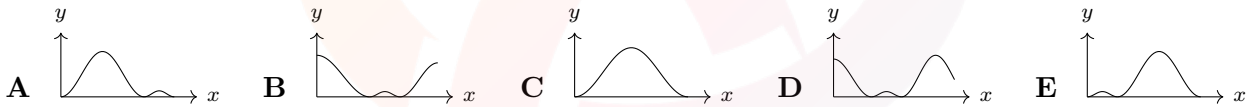
- A** 9 **B** 10 **C** 11 **D** 12 **E** 13

Solution. For $x < 27$, $\lfloor x^{\frac{1}{3}} \rfloor \leq 2$ and $\lfloor x^{\frac{1}{2}} \rfloor \leq 5$, so the sum is at most 7. For $27 \leq x \leq 35$, we get $5 + 3 = 8$, and for $x \geq 36$ the sum is at least $6 + 3 = 9$. Since the sum never decreases as x increases, the solutions are exactly $x = 27, 28, \dots, 35$, which is 9 values.

Answer: A

Question 20

Which of the following is a sketch of the graph of $y = 1 + \sin x \cos x - \sin x - \cos x$ for $0^\circ \leq x \leq 360^\circ$?

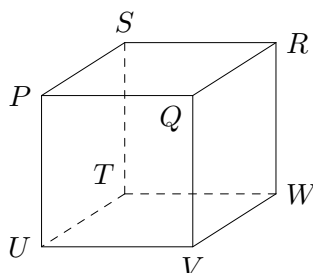


Solution. The expression invites us to perform *Simon's Favourite Factoring Trick*: $y = 1 - \sin x - \cos x + \sin x \cos x = (1 - \sin x)(1 - \cos x)$. Both factors are non-negative, so $y \geq 0$, with zeros only at $x = 0^\circ, 90^\circ$ and 360° . For $0^\circ < x < 90^\circ$ both factors are less than 1, so we only get a small bump (e.g. $y = \left(1 - \frac{\sqrt{2}}{2}\right)^2$ at 45°), whereas for $90^\circ < x < 360^\circ$ the graph rises much higher, e.g. $y = \left(1 + \frac{\sqrt{2}}{2}\right)^2$ at 225° . So we want a small hump followed by a large one, which is E.

Answer: E

Question 21

The diagram shows a clear plastic cube, $PQRSTU VW$. An ant starts at vertex S and walks on the surface of the solid cube. It travels along straight line segments (first to Q , U or W), alternately across the diagonal of a face and then along an edge. An example of a possible five-segment journey is $SQRTSW$. At the end of a journey of 81 segments, which of the following vertices is it impossible for the ant to reach?



A S

B Q

C U

D W

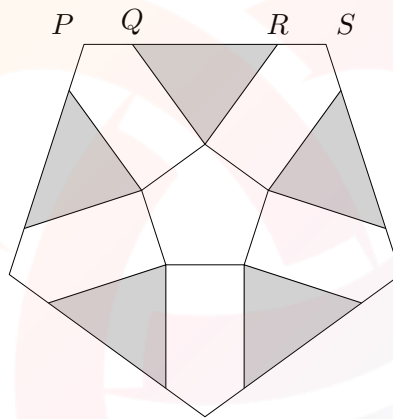
E V

Solution. Colour the vertices black and white so that the two ends of every edge have different colours. Then S, Q, U, W share one colour and P, R, T, V share the other. A face diagonal joins two vertices of the same colour, while an edge swaps the colour. A journey of 81 segments consists of 41 face diagonals and 40 edges, so the colour changes an even number of times, and the ant must finish on one of S, Q, U, W . Hence it can never end at V .

Answer: E

Question 22

As shown in the diagram, a playground is built in the shape of a regular pentagon. In the centre is a smaller regular pentagon, with lines perpendicular to each side at each vertex of the smaller pentagon. The order of the rotational symmetry of the diagram is 5. The side-length of the larger pentagon is 10 m, and $PQ = RS = 2$ m. What fraction of the area of the playground is the area of the unshaded region?



A $\frac{3}{5}$

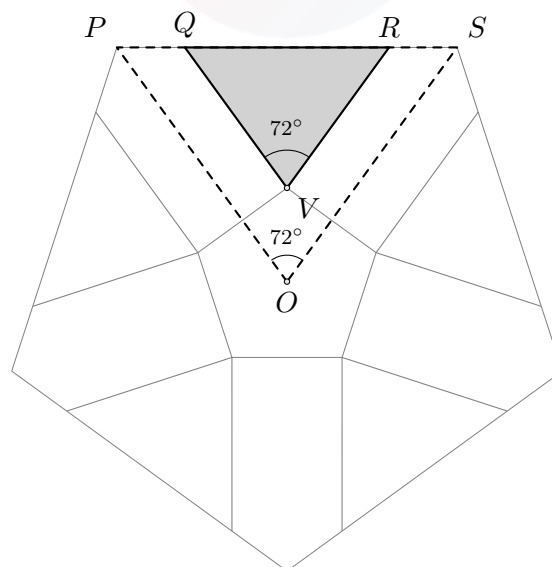
B $\frac{5}{8}$

C $\frac{9}{25}$

D $\frac{16}{25}$

E $\frac{81}{125}$

Solution.



Let O be the common centre and focus on the shaded triangle at the top, whose apex V is the top vertex of the small pentagon. As VQ and VR are perpendicular to the two sides at V , which

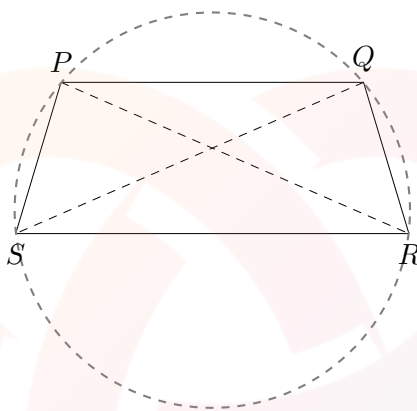
meet at 108° , we get $\angle QVR = 360^\circ - 90^\circ - 90^\circ - 108^\circ = 72^\circ$. Also $QR = 10 - 2 - 2 = 6$ and by symmetry $VQ = VR$. Now $\triangle OPS$, formed by the centre and the top side of the playground, is also isosceles with apex angle 72° , so $\triangle VQR \sim \triangle OPS$ with ratio $\frac{6}{10}$. Hence each shaded triangle has area $\frac{9}{25}[\triangle OPS]$, and since the playground is made of five identical copies of $\triangle OPS$, the unshaded region is $\frac{16}{25}$ of the playground.

Answer: D

Question 23

Let $PQRS$ be a trapezium with QP parallel to RS and with $QP = 20$, $RS = 26$ and $PS = QR = n$ for some positive integer n . The length of the diagonal QS is also a positive integer. What is the maximum possible value of n ?

Solution.

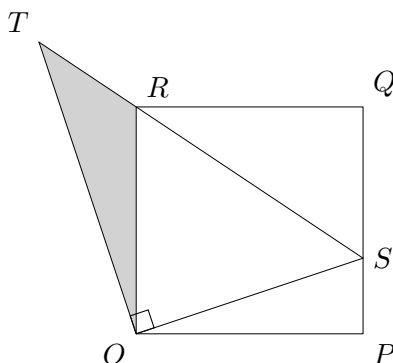


Since $PS = QR$, the trapezium is isosceles and hence cyclic, so by *Ptolemy's Theorem*, $QS \cdot PR = QP \cdot RS + PS \cdot QR$. As $PR = QS$, this gives $QS^2 = 520 + n^2$. Letting $QS = m$, we need $(m - n)(m + n) = 520$. Both factors have the same parity, so they are both even, and to maximise n we take $m - n$ as small as possible, i.e. $m - n = 2$ and $m + n = 260$. This gives $n = 129$ and $m = 131$, and such a trapezium exists since $n > \frac{26-20}{2}$.

Answer: 129

Question 24

$OPQR$ is a square. The point S lies on PQ such that $PS : SQ = 1 : 2$. Points S , R and T lie on a straight line and angle TOS is a right angle. The area of the triangle ORT is 30. What is the area of $OPQR$?



Solution.

We assign $PS = 1$ to find side-length ratios easier. Clearly, $OP = 3$, so by Pythagoras, $OS = \sqrt{3^2 + 1^2} = \sqrt{10}$. Also, $QS = 2$, so $RS = \sqrt{2^2 + 3^2} = \sqrt{13}$. Now, we use the *Law of Cosines* in $\triangle ORS$:

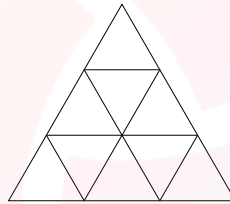
$$\cos \angle OSR = \frac{13 + 10 - 3^2}{2 \cdot \sqrt{10} \cdot \sqrt{13}} = \frac{7}{\sqrt{130}}.$$

Moreover, note that in Rt $\triangle TOS$, $\cos \angle OSR = \frac{OS}{TS} = \frac{\sqrt{10}}{\sqrt{13} + RT}$. Solving, we get $RT = \frac{3\sqrt{13}}{7}$, hence $\frac{TR}{RS} = \frac{3}{7}$, so we easily obtain $[\triangle OSR] = \frac{30}{3} \cdot 7 = 70$. Hence the area of $OPQR$ is simply double that, which is 140.

Answer: 140

Question 25

This diagram, built from equilateral triangles of side-length 1, has a total of 18 edges of length 1. Each edge is to be coloured red, white or blue. Note that different colourings of the edges of the diagram which can be made the same by rotation or reflection of the diagram are to be considered as different colourings. How many different coloured diagrams can be made, given that each equilateral triangle of side-length 1 must have edges of all three colours?



Solution. The six upward-pointing unit triangles use every edge exactly once, while each of the three downward-pointing triangles D_1, D_2, D_3 takes its edges from three different upward ones. So we first colour the edges of the D_i , then complete the upward triangles.

Each pair D_i, D_j has one edge each in a common middle upward triangle, and each D_i has one remaining edge lying in a corner upward triangle. For a middle upward triangle, its two edges from the D 's must differ, and then its third edge is forced. For a corner upward triangle, one edge is already fixed and the other two can be arranged in 2 ways, giving a factor of 2^3 .

Now let x_{ij} be the colour of the edge of D_i lying in the upward triangle shared with D_j . Then D_i is determined by x_{ij} and x_{ik} (its corner edge takes the third colour), and the conditions are $x_{ij} \neq x_{ik}$ and $x_{ij} \neq x_{ji}$. Arranging the six variables in the cyclic order $x_{12}, x_{21}, x_{23}, x_{32}, x_{31}, x_{13}$, these say exactly that neighbouring terms are different, i.e. we are colouring a 6-cycle with 3 colours. By the well-known formula $(k-1)^n + (-1)^n(k-1)$, there are $2^6 + 2 = 66$ ways, so the answer is $66 \cdot 2^3 = 528$.

Answer: 528