

International Mathematical Olympiad

Shortlist 2025 G1-3 Solutions

Kevin Hu

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Overview

This document contains original, rigorous solutions to the International Mathematical Olympiad (IMO) Shortlist 2025. The solutions presented herein emphasize concise, elegant problem-solving techniques native to high-level Olympiad mathematics, avoiding heavy computational bash where synthetic, combinatorial, or purely algebraic insights offer a cleaner path.

Author's Note: Because these are original write-ups and not extracted from official mark schemes, they may contain minor oversights and/or typos. Readers are encouraged to review the arguments critically and explore alternative geometric configurations.

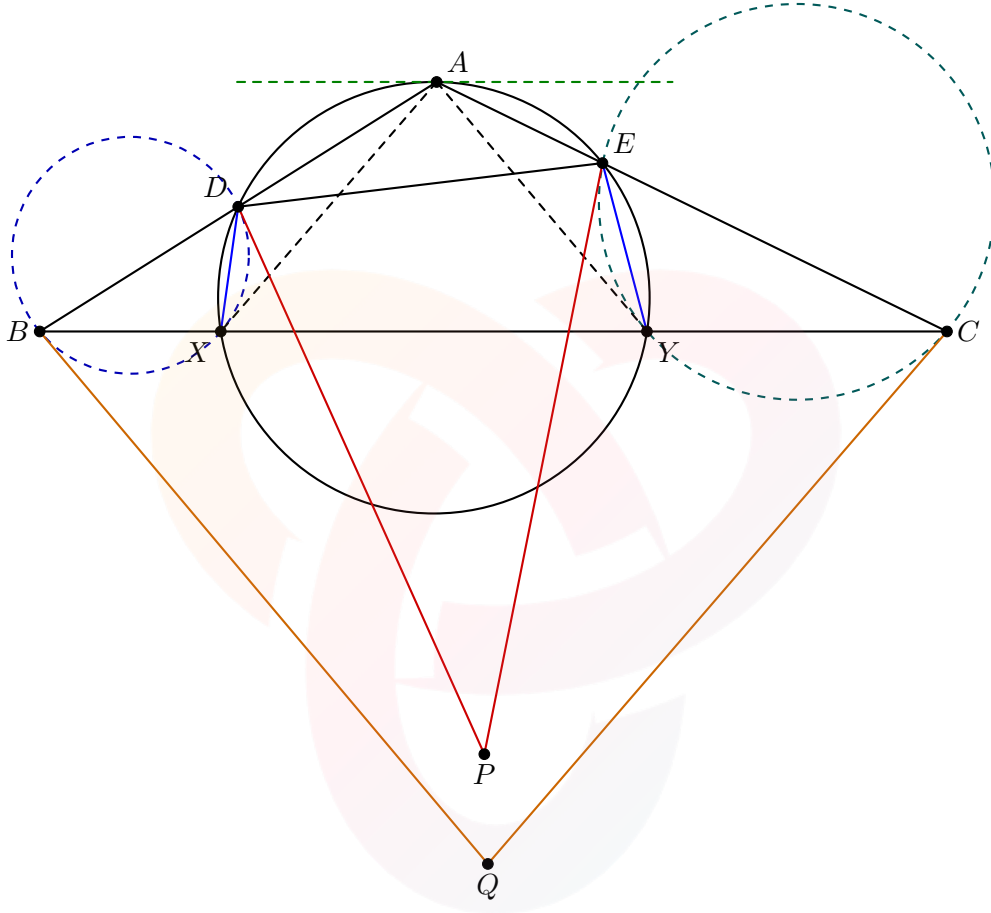
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Problem G1

Let ABC be a triangle such that $\angle A$ is obtuse. Let D and E be points on sides AB and AC , respectively, such that $BDEC$ is cyclic. Suppose the circumcircle of triangle ADE intersects side BC at two points X and Y , where B, X, Y and C lie in that order. Let the circumcircles of triangles BDX and CEY be Ω and Γ , respectively. Let P be a point such that PD is tangent to Ω and PE is tangent to Γ . Let Q be a point such that QB is tangent to Ω and QC is tangent to Γ . Prove that points A, P and Q are collinear.



Solution:

Claim

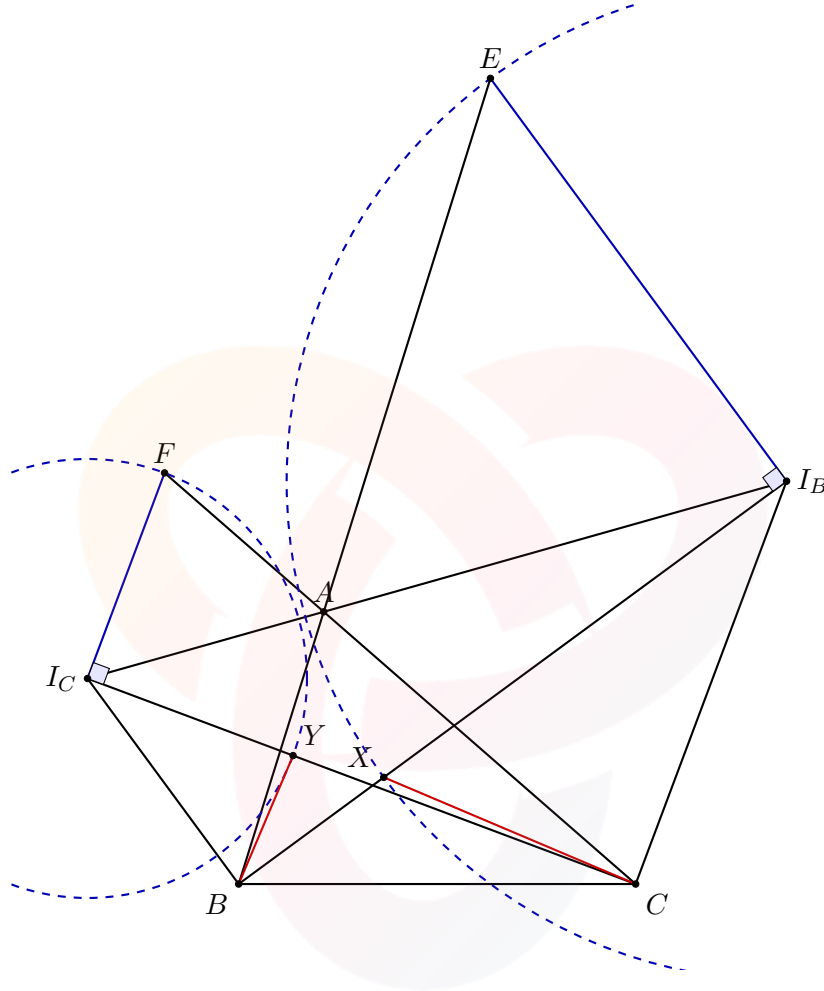
$\triangle AXY$ is isosceles.

Proof. Consider tangent ℓ to $\odot(ADE)$ at point A . By directed angles modulo π , we have $\angle(\ell, AE) = \angle(DA, DE) = \angle(CB, CA)$, hence $\ell \parallel BC$. Now, by symmetry, we have $AX = AY$, which implies the result. $\square$

Since $ADXY$ and $AEYX$ are cyclic, we have $\triangle BDX \sim \triangle BYA$ and $\triangle CYE \sim \triangle CAX$. Thus $\angle BDX = \angle CEY$. By the alternate segment theorem on Ω and Γ , it follows that $\angle XBQ = \angle YCQ$, hence $QB = QC$. Also, by alternate segment theorem, note that $\angle PDE = \pi - \angle ADE - \angle XDP - \angle BDX = \pi - \angle C - \angle BDX - \angle B = \pi - \angle AED - \angle CEY - \angle PEY = \angle PED$, hence $PD = PE$. Thus $Pow(P, \Gamma) = Pow(P, \Omega)$ and $Pow(Q, \Gamma) = Pow(Q, \Omega)$, hence both P and Q lie on the radical axes of Ω and Γ . A does too, since $AD \cdot AB = AE \cdot AC$ as $BDEC$ is cyclic. Therefore, points A, P, Q are collinear. $\blacksquare$

Problem G2

Let ABC be an acute-angled triangle. Let I_B and I_C be the excenters of triangle ABC opposite to B and C , respectively. Let E be a point on the line AB such that $I_BE \perp I_BB$, and let F be a point on the line AC such that $I_CF \perp I_CC$. Suppose the circumcircle of triangle $I_BE B$ intersects the segment I_BB again at X , and the circumcircle of triangle $I_C F C$ intersects the segment $I_C C$ again at Y . Prove that the lines BY and CX are perpendicular.



Solution:

Claim 1

Quadrilateral $I_B I_C B C$ is cyclic.

Proof. As I_B, I_C are excenters, it follows that $\angle B I_C C = \pi - \angle B C I_C - \angle I_C B C = 180^\circ - \frac{C}{2} - (90^\circ + \frac{B}{2}) = 90^\circ - \frac{B}{2} - \frac{C}{2}$. Similarly, $\angle B I_B C = 90^\circ - \frac{B}{2} - \frac{C}{2}$, hence $\angle B I_C C = \angle B I_B C$, meaning $I_B I_C B C$ is cyclic. $\square$

Claim 2

$\triangle B I_C F \sim \triangle E I_B C$

Proof. The perpendiculars imply $I_C F \parallel I_B C$ and $I_C B \parallel I_B E$, which means the angles between the two lines: $\angle B I_C F = \angle E I_B C$. Moreover,

$$\frac{I_C B}{I_B E} = \frac{I_C A}{I_B A} = \frac{I_C F}{I_B C}$$

which proves the result. $\square$

Now, since $I_C Y = I_C F$ and $I_B X = I_B E$, it follows that

$$\frac{I_C Y}{I_B C} = \frac{I_C B}{I_B X},$$

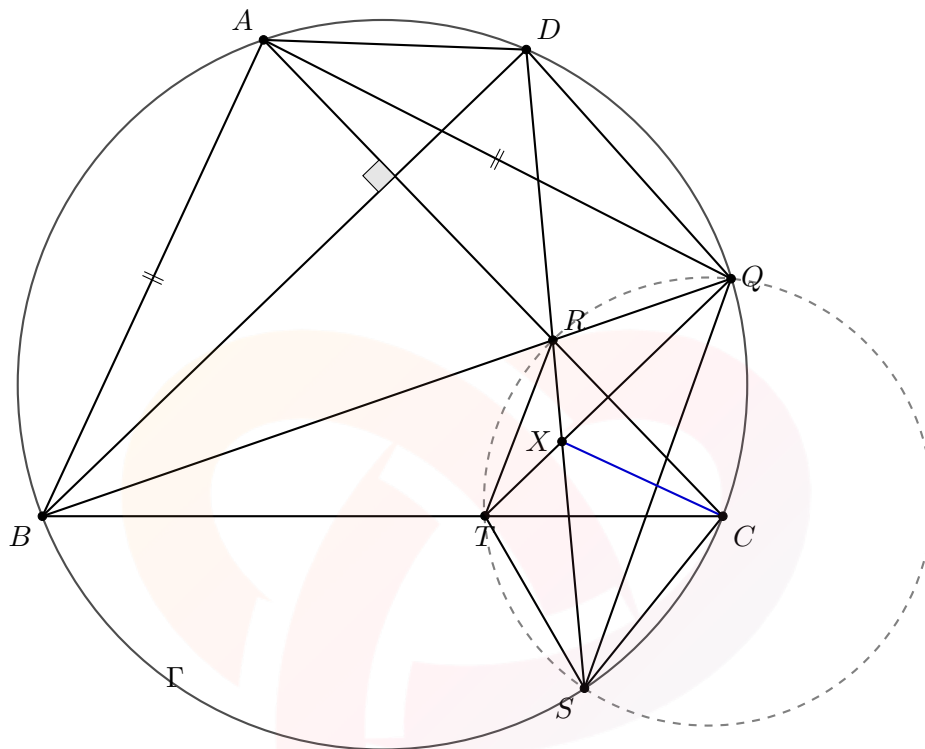
which combined with $\angle B I_C Y = \angle X I_B C$ yields the similarity.

Finally, we will show the angle between corresponding sides of $\triangle B I_C Y$ and $\triangle X I_B C$, which is trivial since $\angle(BY, CX) = \angle(I_C B, X I_B) = 90^\circ$, and we are done. ■



Problem G3

Let ABC be an acute-angled triangle with $AB < AC$. Let Γ be the circumcircle of triangle ABC . The perpendicular from B to AC intersects Γ again at $D \neq B$. Let Q be the point on Γ such that $Q \neq B$ and $AB = AQ$. Lines BQ and AC intersect at R . Line DR intersects Γ again at $S \neq D$. Segment BC intersects the circumcircle of triangle QRS at T . Lines TQ and RS intersect at X . Prove that CX is perpendicular to AB .



Solution:

Claim 1

AC is the perpendicular bisector of segment QT .

Proof. Let T' be the reflection of Q across AC . Since $AB = AQ$, $\angle ACQ = \angle ACB$. Reflection across AC gives $\angle ACT' = \angle ACQ = \angle ACB$, placing T' on line BC . Since $QT' \perp AC$ and $BD \perp AC$, we have $BD \parallel QT'$. Thus, $\angle RQT' = \angle DBQ = \angle RSQ$. Since $R \in AC$, $\triangle RQT'$ is isosceles ($RQ = RT'$), giving $\angle RT'Q = \angle RQT' = \angle RSQ$. Hence, Q, R, S, T' are concyclic. Therefore, $T' = BC \cap \odot(QRS) = T$, hence our phantom point coincides with T . $\square$

Claim 2

$CXTS$ is a cyclic quadrilateral.

Proof. By simple angle chasing and noting that $BD \parallel TQ$ as they are both perpendicular to AC , we get $\angle XSC = \angle DSC = \angle DAC = \angle DBC = \angle XTC$, which implies the result. $\square$

Now, cyclic quadrilaterals $CXTS$ and $QRST$ yield $\angle XCB = \angle XST = \angle RQT$. In $\triangle ABR$, $\angle BAR = \angle A$ and $\angle ABR = \angle C$ (since $AB = AQ$), making the third angle $\angle ARB = \angle B$. Since $TQ \perp AC$, $\angle RQT = 90^\circ - \angle ARB = 90^\circ - \angle B$. Thus, $\angle XCB + \angle B = 90^\circ$, which immediately implies $CX \perp AB$. $\blacksquare$