

HU Collection

ESAT Mock Set E: Mathematics 2

Worked Solutions

Kevin Hu 28 October 2026

Answer key

Q	1	2	3	4	5	6	7	8	9	10
Ans	B	C	F	E	B	A	A	E	B	F

Q	11	12	13	14	15	16	17	18	19	20
Ans	D	D	C	D	A	B	D	A	E	C

Q	21	22	23	24	25	26	27			
Ans	C	B	C	F	E	A	F			

Each solution gives an intended route and names the trap the question was built around. Diagrams show the configuration each solution works with.

Question 1. Answer B.

Write $a = \log_2 x$, $b = \log_2 y$, $c = \log_2 z$. Since $\log_4 x = \frac{1}{2}a$ and $\log_8 x = \frac{1}{3}a$ (and similarly for y and z), the equations become

$$a + \frac{1}{2}b + \frac{1}{3}c = 2, \quad \frac{1}{2}a + \frac{1}{3}b + c = 2, \quad \frac{1}{3}a + b + \frac{1}{2}c = 2.$$

There is no need to solve them: each of a, b, c appears once with each coefficient, so adding gives $\frac{11}{6}(a + b + c) = 6$, that is $a + b + c = \frac{36}{11}$. Hence $\log_2(xyz) = \frac{36}{11}$ and $xyz = 2^{36/11}$.

Treating every logarithm as base 2 gives 2^2 ; solving for a, b, c one at a time is correct but slow.

Question 2. Answer C.

The polynomial $q(x) = p(x) - 2x$ is also a monic quartic, and it vanishes at $x = 1, 2, 3, 4$. By the factor theorem $q(x) = (x-1)(x-2)(x-3)(x-4)$, with no other constant factor because q is monic. So

$$p(6) = 5 \times 4 \times 3 \times 2 + 2 \times 6 = 120 + 12 = 132.$$

Continuing the pattern 2, 4, 6, 8 gives 12, which ignores the quartic term entirely; forgetting to add back $2x$ gives 120.

Question 3. Answer F.

Track what happens to $y = f(x)$. In I, translating by a gives $y = f(x - a)$ and then stretching by k gives $y = kf(x - a)$; in the other order we get $y = kf(x)$ and then $y = kf(x - a)$. The same, because one acts on x and the other on y .

In II both act on x . Translating then stretching by s gives $y = f(\frac{x}{s} - a)$, while stretching then translating gives $y = f(\frac{x-a}{s})$; for $y = x$ these are different lines. Geometrically, the stretch also stretches the translation.

In III, reflecting then stretching gives $y = f(-\frac{x}{s})$, and stretching then reflecting gives the same, since the stretch is centred on the y -axis, which the reflection fixes. So I and III only.

Question 4. Answer E.

Let the first term be a and the ratio r , with $0 < r < 1$. The terms after the first sum to $\frac{ar}{1-r}$, so $a = \frac{3ar}{1-r}$, giving $1-r = 3r$ and $r = \frac{1}{4}$. (The same condition for any later term gives the same equation, after cancelling a power of r .) Then $\frac{a}{1-\frac{1}{4}} = 12$ gives $a = 9$, and the third term is $9 \times \frac{1}{16} = \frac{9}{16}$.

Reading the condition as each term being three times the *next* term gives $r = \frac{1}{3}$, $a = 8$ and third term $\frac{8}{9}$; $\frac{9}{4}$ is the second term.

Question 5. Answer B.

Let $AC = b$. The cosine rule on the side opposite the known angle gives

$$49 = 64 + b^2 - 2 \cdot 8 \cdot b \cos 60^\circ = 64 + b^2 - 8b,$$

so $b^2 - 8b + 15 = 0$ and $b = 3$ or $b = 5$. Both are genuine triangles (sides 8, 7, 3 and 8, 7, 5). Their areas are $\frac{1}{2} \cdot 8 \cdot b \sin 60^\circ = 2\sqrt{3}b$, namely $6\sqrt{3}$ and $10\sqrt{3}$, with sum $16\sqrt{3}$.

The sine rule finds angle C first, and it is easy to keep only the acute value of C and find just the triangle of area $10\sqrt{3}$. Treating the cosine rule as a quadratic in b shows both cases at once.

Question 6. Answer A.

Integrating, $v = 3t^2 - 12t + 9 = 3(t-1)(t-3)$ (using $v = 9$ at $t = 0$) and $s = t^3 - 6t^2 + 9t$ (using $s = 0$ at $t = 0$). The particle reverses at $t = 1$ and at $t = 3$. Now $s(1) = 4$, $s(3) = 0$ and $s(4) = 4$, so it goes out 4 m, back 4 m to O , and out 4 m again: a total distance of 12 m.

The displacement $s(4) = 4$ is the main trap; missing one of the three stages of the motion gives 8.

Question 7. Answer A.

Completing the square, $(x-1)^2 + (y-1)^2 = 9$, so the centre is $C(1, 1)$ and the radius is 3. Then $CP = 6$, and in the right-angled triangle CAP the hypotenuse is twice the side CA , so angle $CPA = 30^\circ$ and the tangent length is $PA = \sqrt{36-9} = 3\sqrt{3}$.

So angle $APB = 60^\circ$ and $PA = PB$: triangle PAB is equilateral with side $3\sqrt{3}$, and its area is $\frac{\sqrt{3}}{4} \cdot 27 = \frac{27\sqrt{3}}{4}$. The kite $PACB$ has area $9\sqrt{3}$, which is the most common wrong answer.

Question 8. Answer E.

The logarithm exists only when $x^2 - 3x > 0$, that is $x < 0$ or $x > 3$. Since $-2 = \log_{1/2} 4$ and $\log_{1/2}$ is a decreasing function, the inequality says $x^2 - 3x < 4$, that is $(x-4)(x+1) < 0$, so $-1 < x < 4$. Combining with the domain gives $-1 < x < 0$ or $3 < x < 4$.

Forgetting the domain leaves $-1 < x < 4$; forgetting that the base is less than 1 reverses the inequality and gives $x < -1$ or $x > 4$.

Question 9. Answer B.

Stationary points of f are the zeros of f' , so there are three, at $x = -1$, 2 and 4, and I is false. Reading signs from the graph: $f' \geq 0$ for $x < 2$ (zero only at -1), $f' < 0$ on $2 < x < 4$ and $f' > 0$ for $x > 4$. So $x = -1$ is a stationary point of inflection, $x = 2$ is a local maximum and $x = 4$ is the only local minimum: II is true.

Since f is increasing on the whole interval $x \leq 2$, $f(-2) < f(0) < f(2)$ and III is true. The usual error is to read the graph as if it were f itself, which suggests a minimum at $x = -1$ and a maximum between 2 and 4.

Question 10. Answer F.

The graph of $y = f(6-2x)$ is obtained from $y = f(x)$ by a translation of 6 to the left (giving $f(x+6)$), then a stretch parallel to the x -axis with scale factor $\frac{1}{2}$ (giving $f(2x+6)$), then a reflection

in the y -axis (giving $f(6 - 2x)$). For $0 \leq x \leq 3$ this covers exactly the original curve for $0 \leq x \leq 6$, reversed and squashed to half the width.

Reflection does not change areas and the stretch halves them, so $\int_0^3 f(6 - 2x) dx = \frac{1}{2}(5 - 3) = 1$, the region below the axis counting negatively. Adding $\int_0^3 2 dx = 6$ gives 7. Forgetting the halving gives 8; adding the two areas instead of subtracting gives 10.

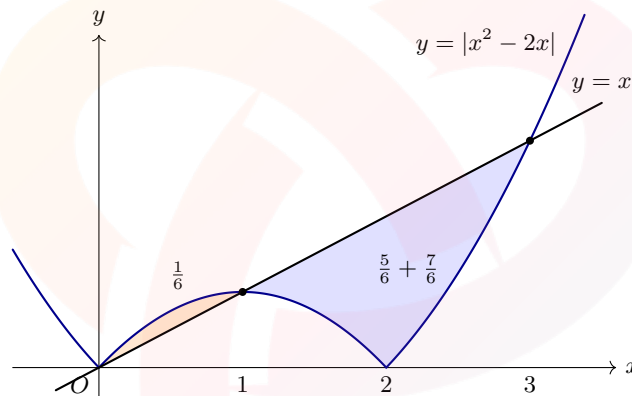
Question 11. Answer **D**.

The sum of the first n terms of an arithmetic progression is a quadratic in n with no constant term: $S_n = An^2 + Bn$. Working in thousands of pounds, $S_{10} = S_{20} = 100$, so the parabola is symmetric about $n = 15$, giving $B = -30A$. Then $S_{10} = 100A - 300A = 100$, so $A = -\frac{1}{2}$ and $S_n = 15n - \frac{1}{2}n^2$.

The greatest value is at $n = 15$: $S_{15} = 225 - 112.5 = 112.5$, that is £112 500. Assuming that the greatest total cannot exceed the two equal totals given gives £100 000.

Question 12. Answer **D**.

The curve is the parabola $y = x^2 - 2x$ with the part between $x = 0$ and $x = 2$ reflected above the axis. It meets $y = x$ where $x^2 - 2x = x$ (so $x = 0$ or 3) and where $2x - x^2 = x$ (so $x = 0$ or 1).



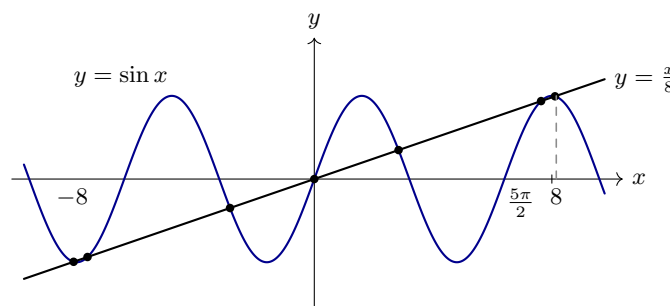
For $0 \leq x \leq 1$ the curve is above the line, and for $1 \leq x \leq 3$ the line is above, giving two regions. Their areas are

$$\int_0^1 (x - x^2) dx + \int_1^2 (x^2 - x) dx + \int_2^3 (3x - x^2) dx = \frac{1}{6} + \frac{5}{6} + \frac{7}{6} = \frac{13}{6}.$$

Ignoring the modulus gives $\int_0^3 (3x - x^2) dx = \frac{9}{2}$; missing the small region near the origin gives 2; using one signed integral gives $\frac{11}{6}$.

Question 13. Answer **C**.

Count intersections of $y = \sin x$ with the line $y = \frac{x}{8}$. Since $|\sin x| \leq 1$, solutions satisfy $|x| \leq 8$, and both sides are odd functions, so solutions come in pairs $\pm x$ together with $x = 0$.



For $x > 0$: near 0 the sine curve rises faster than the line (gradient 1 against $\frac{1}{8}$), and it comes back down to 0 at $x = \pi$, so there is one crossing in $(0, \pi)$. On $(\pi, 2\pi)$ the sine is negative and there are none. On $(2\pi, 8)$ the sine rises to 1 at $x = \frac{5\pi}{2} \approx 7.85$, where the line is only $\frac{5\pi}{16} < 1$; at $x = 8$ the line has reached 1, while $\sin 8 < 1$. So there are two crossings here, and since $8 < 3\pi$ that is all.

That gives 3 positive solutions, 3 negative ones and $x = 0$: 7 in total. The delicate point is that $\frac{5\pi}{2} < 8$; assuming the line is too high by then gives 5.

Question 14. Answer **D**.

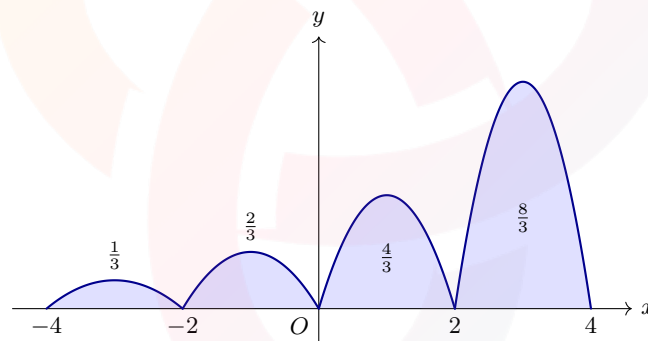
Follow a single point with x -coordinate p . Reflecting in $x = 1$ sends p to $2 - p$, and reflecting in $x = 3$ sends q to $6 - q$. So A then B sends p to $6 - (2 - p) = p + 4$: the pair A, B is a translation of 4 units in the positive x -direction.

The three pairs translate the curve 12 units to the right, giving $y = 2^{x-12}$, and the final A replaces x by $2 - x$, giving $y = 2^{(2-x)-12} = 2^{-10-x}$. It meets the y -axis at $(0, 2^{-10})$.

Taking the pair to be a translation to the left gives 2^{14} ; forgetting the final A gives 2^{-12} ; thinking that two reflections cancel leaves just one reflection and the answer 4.

Question 15. Answer **A**.

The rule says that the graph over each interval of length 2 is the graph over the previous interval, translated 2 to the right and stretched parallel to the y -axis with scale factor 2. Going to the left, each arch is half as tall as the one to its right.



A stretch parallel to the y -axis multiplies areas by its scale factor. The basic arch has area $\int_0^2 (2x - x^2) dx = \frac{4}{3}$, so the arches on $[-4, -2]$, $[-2, 0]$, $[0, 2]$, $[2, 4]$ have areas $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}$, a geometric progression with sum $\frac{15}{3} = 5$.

Treating f as periodic gives $\frac{16}{3}$; doubling to the left as well as to the right gives 12.

Question 16. Answer **B**.

Let the stationary points be at $x = \alpha$ and $x = \beta$ with $\alpha < \beta$; the local maximum comes first because the x^3 coefficient is positive. Then $\frac{dy}{dx} = 3(x - \alpha)(x - \beta)$, so

$$y(\beta) - y(\alpha) = \int_{\alpha}^{\beta} 3(x - \alpha)(x - \beta) dx = -\frac{1}{2}(\beta - \alpha)^3.$$

This equals -32 , so $\beta - \alpha = 4$, and the gradient of the line is $\frac{-32}{4} = -8$. Nothing else about the cubic needs to be found. Dividing 32 by 2 instead of by 4 gives -16 , and dividing by $(\beta - \alpha)^2$ gives -2 . The sign must be negative, since the maximum is on the left.

Question 17. Answer **D**.

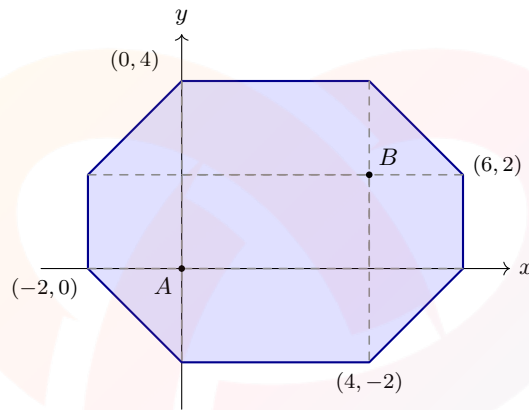
For a cubic the third differences of values at equally spaced inputs are constant. Here the differences are $-1, -3, 3, 9, 27$, then $-2, 6, 6, 18$, then $8, 0, 12$, which are not constant.

An error e in one entry changes the third differences that use it by $e, -3e, 3e, -e$ in turn. The pattern $8, 0, 12$ fits an error in the entry at $x = 3$, which is used by all three third differences with weights $+1, -3, +3$: we need $c + e = 8$, $c - 3e = 0$, $c + 3e = 12$, giving $c = 6$ and $e = 2$. So the output at $x = 3$ is 2 too large and should be 4. Check: $y = x^3 - 4x^2 + 2x + 7$ gives $7, 6, 3, 4, 15, 42$.

Getting the sign of e wrong gives 8. Assuming that the last entry is the faulty one, which is tempting because 42 looks large, does not produce a consistent cubic.

Question 18. Answer A.

The condition separates into an x -part and a y -part: $(|x| + |x - 4|) + (|y| + |y - 2|) \leq 10$. The first bracket is 4 for $0 \leq x \leq 4$ and is $4 + 2u$ when x is a distance u outside that interval; likewise the second is $2 + 2v$, where v is the distance of y outside $[0, 2]$. So the condition is $6 + 2u + 2v \leq 10$, that is $u + v \leq 2$.

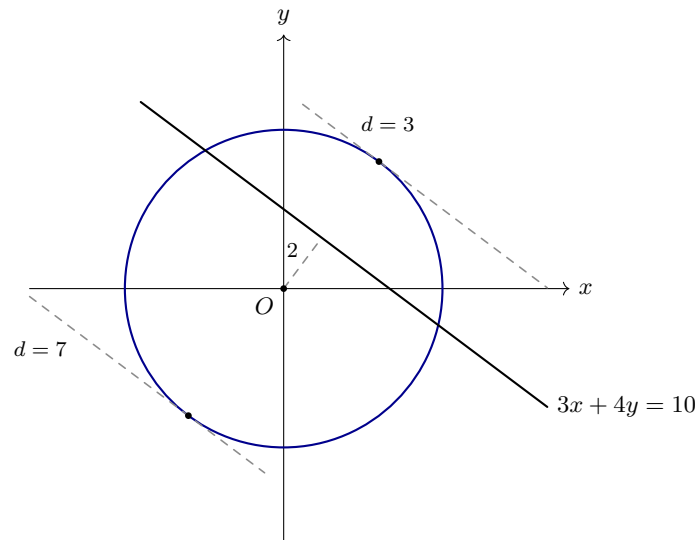


The region is the 4×2 rectangle, with 2×4 strips above and below, 2×2 strips at the sides, and in each corner a right-angled triangle with legs 2: area $8 + 16 + 8 + 4 \times 2 = 40$.

Filling the corners with squares gives 48; leaving them out gives 32; forgetting the factor 2 (so that $u + v \leq 4$) gives 88.

Question 19. Answer E.

The centre O is at distance $\frac{|0+0-10|}{5} = 2$ from the line, and the radius is 5. So points of the circle on the same side as O have distances from 0 up to $2 + 5 = 7$, and points on the far side have distances from 0 up to $5 - 2 = 3$.



For each distance d , the points at distance d lie on the two lines parallel to the wall at distance d from it. Distance 0: the wall itself is a chord, 2 points. Distances 1 and 2: both parallels are chords, 4 points each. Distance 3: the far parallel is at distance 5 from O and is a tangent, so $1 + 2 = 3$ points. Distances 4, 5, 6: only the near parallel meets the circle, 2 points each. Distance 7: that parallel is a tangent, 1 point.

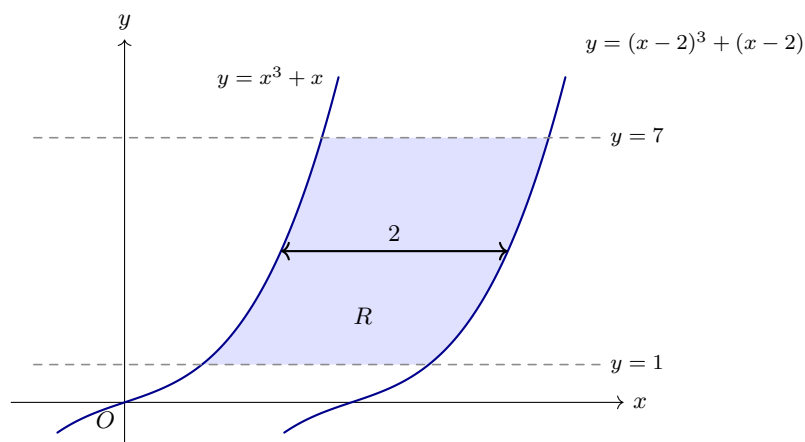
Total $2 + 4 + 4 + 3 + 6 + 1 = 20$. Counting 2 points at each tangent gives 22; forgetting the far side gives 16.

Question 20. Answer C.

The natural method needs the x -coordinates where each curve meets $y = 1$ and $y = 7$, which are roots of awkward cubics. The insight is that

$$(x - 2)^3 + (x - 2) = x^3 - 6x^2 + 13x - 10,$$

so the second curve is the first translated 2 units in the positive x -direction. Both curves are increasing, so every horizontal line between $y = 1$ and $y = 7$ crosses R in a segment of length exactly 2.



Slicing R horizontally (or cutting it into pieces and sliding them, as with a parallelogram), its area is $2 \times (7 - 1) = 12$. Using the vertical separation of the curves, or a width of 1, leads to the other options.

Question 21. Answer C.

The gradient of C_n is $1 + x + x^2 + \dots + x^{n-1}$, a geometric series, equal to $\frac{1 - x^n}{1 - x}$ for $x \neq 1$ and to n at $x = 1$. It is zero exactly when $x^n = 1$ with $x \neq 1$.

For $n = 20$ this means $x = -1$ only, so I is true. For $n = 21$ the only real solution of $x^{21} = 1$ is $x = 1$, where the gradient is 21, so there are no stationary points and II is false.

For $n = 10$ and $-1 < x < 1$, both $1 - x^{10}$ and $1 - x$ are positive; for $x > 1$ both are negative; and at $x = 1$ the gradient is 10. So the gradient is positive for all $x > -1$ and III is true. Answer: I and III only.

Question 22. Answer B.

Working out trapezium sums for large n is hopeless, so look at one strip. On a strip from $x = a$ to $x = a + h$, the trapezium has area $\frac{h}{2}(a^2 + (a + h)^2) = a^2h + ah^2 + \frac{1}{2}h^3$, while the exact area is $\frac{1}{3}((a + h)^3 - a^3) = a^2h + ah^2 + \frac{1}{3}h^3$. The excess is $\frac{1}{6}h^3$, the same for every strip.

With n strips, $h = \frac{3}{n}$ and the total excess is $n \cdot \frac{1}{6} \cdot \frac{27}{n^3} = \frac{9}{2n^2}$. We need $\frac{9}{2n^2} < 0.01$, that is $n^2 > 450$. Since $21^2 = 441$ and $22^2 = 484$, the answer is 22.

Using $\frac{1}{12}h^3$ per strip (the area between a chord and a parabola is often misremembered) gives $\frac{9}{4n^2} < 0.01$ and the answer 16.

Question 23. Answer C.

Negative x : here $0 < 2^x < 1$. If n is odd then $x^n < 0$, so there are no solutions. If n is even, x^n decreases from very large values to 0 as x goes from very negative values to 0, while 2^x increases, so there is exactly one solution.

Positive x : for $0 < x \leq 1$ we have $x^n \leq 1 < 2^x$, so there are none. For $n = 1$, $2^x > x$ for all x , so $N(1) = 0$. For $n = 2$ there are the solutions $x = 2$ and $x = 4$ (and no others: $2^{x/2}$ and x cross at most twice). For $n \geq 3$, x^n is below 2^x at $x = 1$, above it at $x = 2$ (as $2^n > 4$), and below it again for very large x , since an exponential eventually beats any power; comparing $\frac{x}{\log_2 x}$ with n shows there are exactly two crossings.

So $N(1) = 0$, $N(n) = 3$ for even n and $N(n) = 2$ for odd $n \geq 3$: the total is $0 + 5 \times 3 + 4 \times 2 = 23$. Forgetting the negative solutions gives 18; giving odd n a negative solution too gives 27.

Question 24. Answer F.

The coefficients read the same in both directions, and $x = 0$ is not a root, so divide by x^2 :

$$(x^2 + \frac{1}{x^2}) + 4(x + \frac{1}{x}) + q = 0.$$

With $t = x + \frac{1}{x}$ this is $t^2 + 4t + (q - 2) = 0$, so $t = -2 \pm \sqrt{6 - q}$. Each value of t with $|t| > 2$ gives two distinct real x , $t = \pm 2$ gives one ($x = \pm 1$), and $|t| < 2$ gives none.

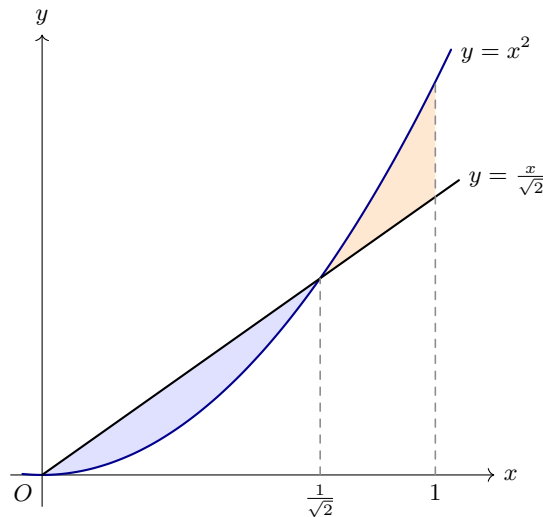
Let $s = \sqrt{6 - q}$. If $s > 0$, the root $t = -2 - s$ is always below -2 and gives two roots. The other root $t = -2 + s$ gives none when $0 < s < 4$, one root ($x = 1$) when $s = 4$, and two more when $s > 4$. If $s = 0$ the only root is $x = -1$. So there are exactly two distinct real roots exactly when $0 < 6 - q < 16$, that is $-10 < q < 6$: the 15 integers from -9 to 5 .

Including $q = -10$ (three distinct roots, one of them double) gives 16; including $q = 6$ as well gives 17.

Question 25. Answer E.

If $m \leq 0$ the misfit is $\int_0^1 (x^2 - mx) dx \geq \frac{1}{3}$, and if $m \geq 1$ it is $\int_0^1 (mx - x^2) dx \geq \frac{1}{6}$. For $0 < m < 1$ the line is above the curve for $0 < x < m$ and below it for $m < x < 1$, so the misfit is

$$A(m) = \int_0^m (mx - x^2) dx + \int_m^1 (x^2 - mx) dx = \frac{m^3}{3} - \frac{m}{2} + \frac{1}{3}.$$

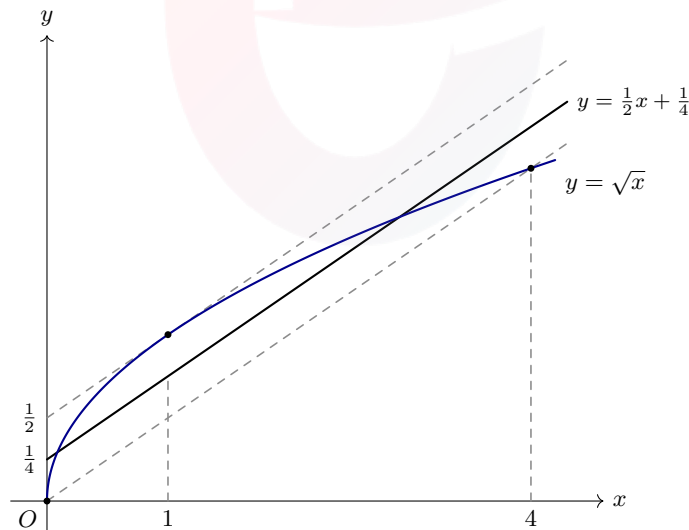


Then $A'(m) = m^2 - \frac{1}{2} = 0$ at $m = \frac{1}{\sqrt{2}}$, a minimum, with $A = \frac{1}{6\sqrt{2}} - \frac{1}{2\sqrt{2}} + \frac{1}{3} = \frac{1}{3} - \frac{\sqrt{2}}{6} = \frac{2-\sqrt{2}}{6} \approx 0.098$, which is less than $\frac{1}{6}$.

The trap is to make the signed integral zero, which gives $m = \frac{2}{3}$ and misfit $\frac{8}{81} \approx 0.099$: very close, but not the least. Taking $m = \frac{1}{2}$ gives $\frac{1}{8}$.

Question 26. Answer **A**.

The chord from $(0,0)$ to $(4,2)$ is $y = \frac{1}{2}x$. The tangent parallel to it touches where $\frac{1}{2\sqrt{x}} = \frac{1}{2}$, at $(1,1)$, and is $y = \frac{1}{2}x + \frac{1}{2}$. The curve lies in the strip between these two lines, which are $\frac{1}{2}$ apart vertically. The line midway, $y = \frac{1}{2}x + \frac{1}{4}$, is within $\frac{1}{4}$ of the curve everywhere, with gap exactly $\frac{1}{4}$ at $x = 0, 1$ and 4 , so $E = \frac{1}{4}$ is possible.



No line does better. Let $g(x) = \sqrt{x} - (mx + c)$. For any line, $3(m \cdot 0 + c) - 4(m + c) + (4m + c) = 0$, so $3g(0) - 4g(1) + g(4) = 3 \cdot 0 - 4 \cdot 1 + 2 = -2$ whatever m and c are. If every gap were less than $\frac{1}{4}$, the left side would have size less than $\frac{3}{4} + 1 + \frac{1}{4} = 2$, which is impossible.

Using the chord or the tangent alone gives $\frac{1}{2}$.

Question 27. Answer **F**.

Work with the coefficients of $(1+x)^{100}$, keeping track only of whether each coefficient is odd or even. Since $(1+x)^2 = 1 + 2x + x^2$, the polynomial $(1+x)^2$ has the same odd coefficients as $1 + x^2$.

Squaring repeatedly, $(1+x)^4$ has the same odd coefficients as $1+x^4$, and so on: $(1+x)^{2^k}$ behaves like $1+x^{2^k}$. (Multiplying polynomials preserves this, because only the parities of products and sums matter.)

Now $100 = 64 + 32 + 4$, so $(1+x)^{100}$ has the same odd coefficients as

$$(1+x^{64})(1+x^{32})(1+x^4).$$

Expanding gives 8 distinct powers $x^0, x^4, x^{32}, x^{36}, x^{64}, x^{68}, x^{96}, x^{100}$, each with coefficient 1. So exactly 8 of the coefficients are odd.

The answer is 2 raised to the number of 1s in the binary form 1100100 of 100. Building Pascal's triangle modulo 2 for 100 rows is far too long.

