

HU Collection

ESAT Mock Set E: Mathematics 1

Worked Solutions

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Answer key

Q	1	2	3	4	5	6	7	8	9	10
Ans	A	E	F	A	E	E	C	B	F	A

Q	11	12	13	14	15	16	17	18	19	20
Ans	B	C	F	A	A	D	C	B	B	F

Q	21	22	23	24	25	26	27			
Ans	D	C	E	C	D	D	B			

Each solution gives an intended route and names the trap the question was built around. Diagrams show the configuration each solution works with.

Question 1. Answer A.

The intensity is multiplied by

$$\frac{1.2}{1.25^2} = 1.2 \times \frac{16}{25} = \frac{19.2}{25} = 0.768,$$

so it falls by 23.2%.

Treating the dependence on distance as inverse rather than inverse square gives $\frac{1.2}{1.25} = 0.96$, a 4% drop; simply combining the percentages gives 5%; ignoring the new lamp gives 36%; and adding the 20% to 0.64 instead of multiplying gives 16%.

Question 2. Answer E.

The ream has mass $500 \times \frac{1}{16} \times 80 = 2500$ g = 2.5 kg and volume $\frac{1}{16} \times 0.05 = \frac{1}{320}$ m³, so its density is $2.5 \times 320 = 800$ kg/m³.

The area of the sheet cancels: one sheet is $\frac{0.05}{500} = 10^{-4}$ m thick, and 80 g/m² divided by 10^{-4} m is 800 000 g/m³. The option 0.8 is the density in g/cm³, not kg/m³.

Question 3. Answer F.

The patient needs $0.5 \times 60 = 30$ mg per hour. The solution contains 250 mg per 100 ml, that is 2.5 mg per ml, so 30 mg is in 12 ml. That is $12 \times 20 = 240$ drops per hour, or 4 drops per minute. Reading 0.25% as 0.25 mg per ml gives 40; the options 12 and 240 are the hourly figures in ml and in drops.

Question 4. Answer A.

Completing the square, $2x^2 - 12x + 23 = 2(x - 3)^2 + 5$, with turning point (3, 5). Moving the turning point to the origin gives $y = 2x^2$; reflecting in the x -axis gives $y = -2x^2$; the final translation gives

$$y = -2(x - 1)^2 + 5.$$

This is zero when $(x - 1)^2 = \frac{5}{2}$, so $x = 1 \pm \sqrt{\frac{5}{2}} = 1 \pm \frac{\sqrt{10}}{2}$. The original curve lies above the x -axis, but the reflection turns it into a downward parabola with maximum 5, so it does cross. Multiplying by 2 instead of dividing gives $1 \pm \sqrt{10}$, and translating the wrong way gives $-1 \pm \frac{\sqrt{10}}{2}$.

Question 5. Answer **E**.

We need $\sin \theta \geq \frac{1}{2}$ with $0 \leq \theta \leq 720$. On the sine graph this happens for $30 \leq \theta \leq 150$ and again, one period later, for $390 \leq \theta \leq 510$.

Since $t = \theta/30$, the ship can pass for $1 \leq t \leq 5$ and $13 \leq t \leq 17$: two spells of 4 hours, so 8 hours in total. The answer 4 uses only one period of the tide, and 16 is the time when the depth is below 6.5 metres.

Question 6. Answer **E**.

The model has volume $\frac{2200}{8.8} = 250 \text{ cm}^3$. Lengths scale by 20, so volumes scale by $20^3 = 8000$ and the statue has volume $2\,000\,000 \text{ cm}^3$. Its mass is $2\,000\,000 \times 2.7 = 5\,400\,000 \text{ g} = 5.4 \text{ tonnes}$. Scaling by the area factor 400 gives 0.27; keeping the density of bronze gives 17.6.

Question 7. Answer **C**.

The frequency of a class is the rise of the cumulative frequency graph across it, so the frequency density is the gradient of that segment. The frequencies are 24, 30, 36, 60, 50, and dividing by the class widths 10, 5, 5, 15, 25 gives densities 2.4, 6, 7.2, 4, 2.

The tallest bar is the steepest segment, $15 \leq t < 20$, with density 7.2. The class $20 \leq t < 35$ has the largest frequency, but in a histogram height is frequency density, not frequency.

Question 8. Answer **B**.

The angle $\angle A_4 A_1 A_5$ is an angle at the circumference standing on the arc $A_4 A_5$ that does not contain A_1 . This arc contains no other labelled point, so the angle is half of the short arc between the neighbours A_4 and A_5 .

In the same way each tip angle is half of one of the seven arcs between consecutive points, and each such arc is used once. The seven arcs make up the whole circle, so the sum is $\frac{1}{2} \times 360^\circ = 180^\circ$, whatever the spacing. The answer 540° is the sum for the fatter star in which each point is joined to the points two places away.

Question 9. Answer **F**.

The first five marks total 360 and the six total 444, so the sixth mark is 84. Write the first five in order as $a \leq b \leq 70 \leq d \leq e$.

The median of six marks is the mean of the third and fourth. Since $84 > 70$, the third is 70 and the fourth is the smaller of d and 84. If $d \geq 84$ the median would be 77, so $d < 84$ and $\frac{70 + d}{2} = 72$, giving $d = 74$. For example, 60, 66, 70, 74, 90 followed by 84 fits every condition.

Question 10. Answer **A**.

The distance is the area under the graph of speed against time. The six trapezia give

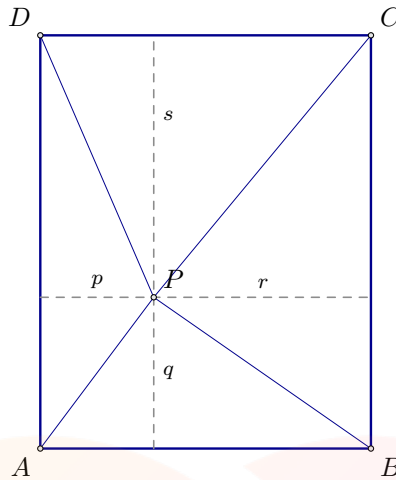
$$10 \left(\frac{0+31}{2} + 12 + 20 + 25 + 28 + 30 \right) = 10(15.5 + 115) = 1305 \text{ m}.$$

The top edge of each trapezium is a chord of the curve, and every chord lies below the curve, so each trapezium misses a sliver of area: 1305 m is an underestimate. The values 1150 and 1460 come from rectangles using left and right end speeds, and 2610 forgets to halve.

Question 11. Answer **B**.

Drop perpendiculars from P to the four sides. If P is at distances p and q from the sides through A , and r and s from the opposite sides, then by Pythagoras

$$PA^2 = p^2 + q^2, \quad PB^2 = r^2 + q^2, \quad PC^2 = r^2 + s^2, \quad PD^2 = p^2 + s^2.$$



So $PA^2 + PC^2 = PB^2 + PD^2$ for opposite corners, and $PD^2 = 2500 + 8100 - 4900 = 5700$, giving $PD = 10\sqrt{57}$ m, about 75.5 m. The dimensions of the field are not needed (and such a field exists, for instance with P at 30 m and 40 m from two sides). Pairing the wrong corners gives $10\sqrt{105}$.

Question 12. Answer **C**.

If the scores in order are $x \leq y \leq z$, the mean equals the median y exactly when $x + z = 2y$, that is when the three scores form an arithmetic sequence (possibly with all three equal). There are $6 \times 6 \times 4 = 144$ equally likely outcomes; call the four-sided score c .

If c is the middle term, the six-sided scores satisfy $a + b = 2c$: this gives 1, 3, 5, 5 ordered pairs for $c = 1, 2, 3, 4$, a total of 14. If c is an end term with common difference $d \geq 1$, the other two are $c + d, c + 2d$ or $c - d, c - 2d$, in either order: $c = 1$ gives $\{2, 3\}, \{3, 5\}$; $c = 2$ gives $\{3, 4\}, \{4, 6\}$; $c = 3$ gives $\{4, 5\}, \{1, 2\}$; $c = 4$ gives $\{5, 6\}, \{2, 3\}$. That is 8 sets, each in 2 orders, so 16 outcomes.

The probability is $\frac{14 + 16}{144} = \frac{5}{24}$. Forgetting the four outcomes with all scores equal gives $\frac{13}{72}$, and treating all three dice as six-sided gives $\frac{7}{36}$.

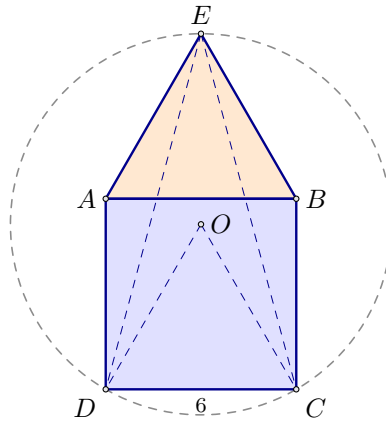
Question 13. Answer **F**.

While they approach each other, and again after they pass, the gap changes at the sum of their speeds: 6 km in 12 minutes is 30 km/h. Once the cyclist has turned at A (the corner at 20 minutes), the gap shrinks at the difference of their speeds: 1 km in 10 minutes is 6 km/h.

So the cyclist rides at 18 km/h and the jogger runs at 12 km/h. At 20 minutes the gap is 4 km, which closes at 6 km/h in 40 minutes, so the catch happens after 1 hour, when the jogger is 12 km from A . (Check: they pass 2.4 km from A at 12 minutes, and the cyclist covers that 2.4 km in 8 minutes.)

Question 14. Answer **A**.

Triangle ADE is isosceles with $AD = AE = 6$, and $\angle DAE = 90^\circ + 60^\circ = 150^\circ$, so $\angle AED = 15^\circ$. By symmetry $\angle BEC = 15^\circ$ too, so $\angle DEC = 60^\circ - 15^\circ - 15^\circ = 30^\circ$.



$\angle DEC$ is an angle at the circumference standing on the chord DC , so the angle at the centre O is $\angle DOC = 60^\circ$. Triangle DOC is isosceles with a 60° apex angle, hence equilateral, and the radius equals $DC = 6$ cm. (Check with coordinates: O is $3\sqrt{3}$ above DC and E is $6 + 3\sqrt{3}$ above it, so $OE = 6$.) The value $3\sqrt{2}$ is the distance from the centre of the square to a corner, which is not equidistant from E .

Question 15. Answer A.

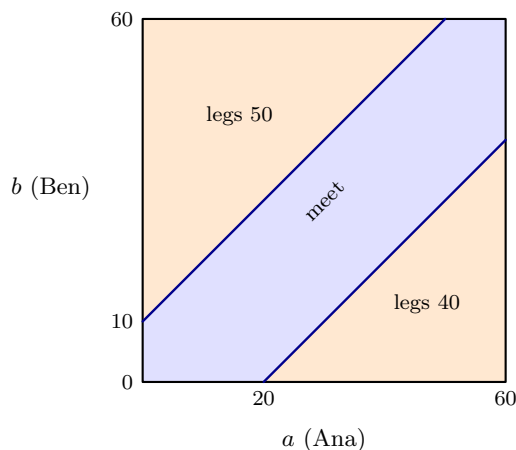
I: $y = f(-x)$ is the reflection of the graph in the y -axis, and the two graphs meet where $f(x) = f(-x)$. This always happens at $x = 0$, and the other meeting points come in pairs $\pm x$. On $0 < x \leq 4$ the mirror image runs through $(0, 1)$, $(2, 3)$ and $(4, -1)$. It crosses the graph of f at $x = \frac{9}{7}$ (both heights are $\frac{16}{7}$) and at $x = \frac{15}{4}$ (both heights are $-\frac{1}{2}$), and nowhere else. With $x = 0$ and the mirror points $-\frac{9}{7}$, $-\frac{15}{4}$, there are 5 points: true.

II: the graphs meet where $f(x) + 1 = -f(x)$, that is $f(x) = -\frac{1}{2}$. The horizontal line $y = -\frac{1}{2}$ crosses the graph at $x = -\frac{15}{4}$, $x = \frac{12}{5}$ and $x = \frac{15}{4}$, which is 3 points, not 2. The crossing on the first segment, between $(-4, -1)$ and $(-2, 3)$, is the one usually missed. False.

III: $y = f(x - 2) - 1$ is the graph moved 2 right and 1 down, defined for $-2 \leq x \leq 4$. Sketching it over the original gives crossings at $x = -\frac{1}{3}$, $\frac{1}{3}$, $\frac{19}{9}$ and $\frac{35}{9}$: 4 points, true. So I and III only.

Question 16. Answer D.

Let Ana and Ben arrive a and b minutes after 1 pm, so (a, b) is a random point of a 60 by 60 square. They meet when $b - a \leq 10$ (if Ana is first) and $a - b \leq 20$ (if Ben is first), that is when the point lies in the band between the lines $b = a + 10$ and $b = a - 20$.



The two corner triangles outside the band have legs 50 and 40, so their total area is $\frac{1}{2}(2500 + 1600) =$

2050. The probability of meeting is

$$1 - \frac{2050}{3600} = \frac{1550}{3600} = \frac{31}{72}.$$

The option $\frac{41}{72}$ is the probability that they do not meet; $\frac{7}{16}$ is the answer if both waited 15 minutes.

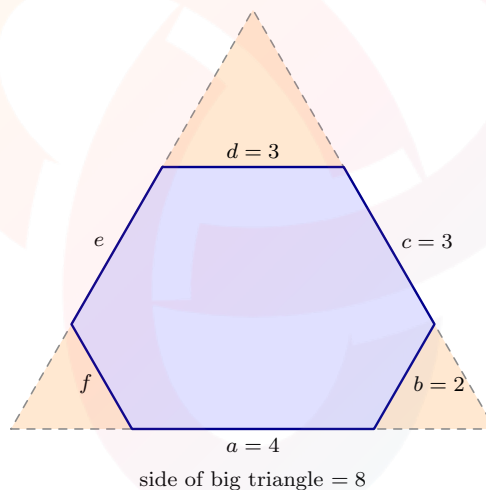
Question 17. Answer **C**.

Write $n = a^2 - b^2 = (a - b)(a + b)$. The two factors $a - b$ and $a + b$ have the same parity, so n is either odd or a multiple of 4; numbers of the form $4k + 2$ are impossible.

An odd $n \geq 3$ works: $n = (k + 1)^2 - k^2$ with $n = 2k + 1$, $k \geq 1$. A multiple of 4 with $n = 4m \geq 8$ works: $n = (m + 1)^2 - (m - 1)^2$ with $m - 1 \geq 1$. The exceptions are 1 and 4, which need $b = 0$. The odd numbers from 3 to 999 number 499 and the multiples of 4 from 8 to 996 number 248, so the total is 747. Allowing $b = 0$ would give 749.

Question 18. Answer **B**.

Extend the first, third and fifth sides. Since every exterior angle is 60° , they form an equilateral triangle, and the hexagon is this triangle with three small equilateral triangles cut off at its corners, whose sides are the second, fourth and sixth sides of the hexagon.



Call the sides a, b, c, d, e, f in order, with $a = 4$, $b = 2$, $c = 3$, $d = 3$. One side of the big triangle is $f + a + b$, another is $b + c + d = 8$ and the third is $d + e + f$. All three equal 8, so $f = 2$ and then $e = 3$.

The area is $\frac{\sqrt{3}}{4} (8^2 - 2^2 - 3^2 - 2^2) = \frac{47\sqrt{3}}{4}$. Assuming the missing sides repeat the pattern, or cutting off the wrong corner triangles, gives the other values.

Question 19. Answer **B**.

A real power B^E equals 1 in three ways: the base is 1; the exponent is 0 and the base is not 0; or the base is -1 and the exponent is an even integer.

Base 1: $x^2 - 5x + 4 = 0$, so $x = 1$ or 4. Exponent 0: $x^2 - 4x - 5 = (x - 5)(x + 1) = 0$, so $x = 5$ or -1 , where the base is 5 and 11, both non-zero. Base -1 : $x^2 - 5x + 6 = 0$, so $x = 2$ or 3. At $x = 2$ the exponent is -9 , odd, giving $(-1)^{-9} = -1$, so $x = 2$ fails; at $x = 3$ the exponent is -8 , even, so $x = 3$ works.

The solutions are $-1, 1, 3, 4, 5$, with sum 12. Including $x = 2$ gives 14, and missing the base -1 case gives 9.

Question 20. Answer **F**.

I: $x = 2$ is a solution, and the exponential curve $y = 2^x$, which tends to 0 on the left, also crosses the line $y = x + 2$ once between $x = -2$ and $x = -1$ (at $x = -2$ the curve is above, at $x = -1$ it is below). Two solutions.

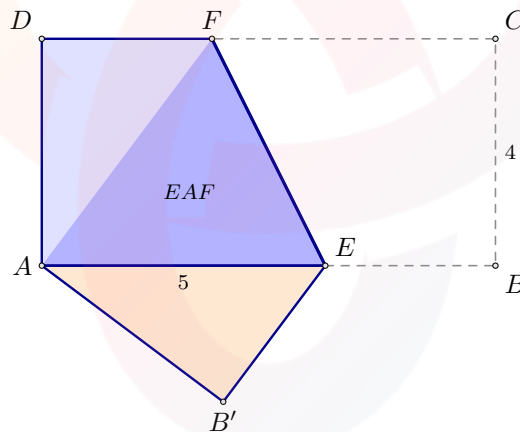
II: the branch of $y = \frac{1}{x}$ for $x > 0$ meets the rising part of the parabola once. For $x < 0$ the branch starts near 0 on the left and plunges to $-\infty$, while the parabola is above the curve at $x = -2$, below it at $x = -1$ and above it again just left of 0; they cross twice (between -2 and -1 , and between -1 and 0). Three solutions.

III: for $x > 0$ a decreasing curve meets an increasing one once. For $x < 0$, $x = -2$ and $x = -4$ both work ($4 = 4$ and $16 = 16$), and at $x = -3$ we have $8 < 9$, so the curves cross at each. Three solutions.

IV: $x = 0$ works and solutions come in pairs $\pm x$. For $x > 0$ the line is below 1 only for $x < 500$. It meets the first hump of the sine curve once in $0 < x < 180$, misses the negative part, and meets the next hump twice in $360 < x < 500$, since at $x = 450$ we have $1 > 0.9$. That makes $3 + 3 + 1 = 7$. So II and III only.

Question 21. Answer **D**.

The crease is the perpendicular bisector of AC . It passes through the centre of the rectangle and meets AB at E and CD at F . Since E is on the bisector, $EA = EC$; writing $EA = x$, Pythagoras in triangle EBC gives $x^2 = (8 - x)^2 + 16$, so $x = 5$.

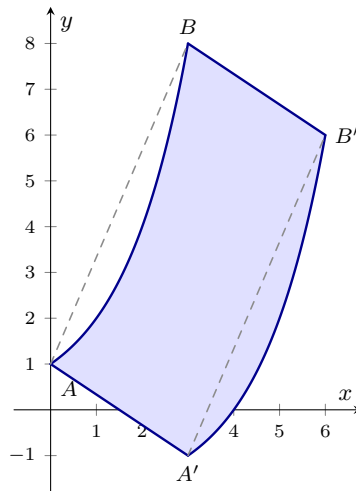


The crease splits the sheet into two congruent trapezia of area 16. When $EBCF$ is folded over, C goes to A and triangle ECF lands on triangle EAF , which is part of the other trapezium; the rest of the folded flap hangs outside the original rectangle. So the covered area is $16 + 16 - [EAF]$.

Triangle EAF has base $EA = 5$ and height 4, area 10. The covered area is 22, which is $\frac{22}{32} = \frac{11}{16}$ of the sheet. Answering $\frac{1}{2}$ assumes the flap lies entirely on top of the other half.

Question 22. Answer **C**.

We cannot integrate 2^x here, and we do not need to. Join A to B and A' to B' by straight chords. The region is the parallelogram $ABB'A'$, plus the sliver between Γ and the chord AB on one side, minus the congruent sliver between Γ' and $A'B'$ on the other side (the second sliver is the first one translated).



So the area equals the area of the parallelogram with sides $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and $\overrightarrow{AA'} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Enclosing it in the rectangle from $x = 0$ to 6 and $y = -1$ to 8, of area 54, and removing two triangles of area $\frac{1}{2} \times 3 \times 7$ and two of area $\frac{1}{2} \times 3 \times 2$, leaves $54 - 21 - 6 = 27$.

The value 21 is what a horizontal translation by 3 alone would give, since then every horizontal cross-section has length 3 over a height of 7.

Question 23. Answer **E**.

Four digits occupy the odd places and three the even places. If their sums are S and T , then $S + T = 28$, so $S - T = 28 - 2T$ is even and lies between $28 - 2 \times 18 = -8$ and $28 - 2 \times 6 = 16$. The only multiples of 11 available are 0, so $T = 14$.

The sets of three digits from 1 to 7 with sum 14 are $\{1, 6, 7\}$, $\{2, 5, 7\}$, $\{3, 4, 7\}$ and $\{3, 5, 6\}$. Each gives $3! \times 4! = 144$ numbers, so there are 576 multiples of 11 among the $7! = 5040$ numbers. The probability is $\frac{576}{5040} = \frac{4}{35}$.

The answer $\frac{1}{11}$ assumes the multiples of 11 are spread evenly; missing one of the four triples gives $\frac{3}{35}$.

Question 24. Answer **C**.

Write $g = \text{HCF}(a, b)$, $a = gx$ and $b = gy$, where x and y have no common factor. Then $\text{LCM}(a, b) = gxy$, so $a \star b = xy$, and the condition is $xy = 36$ with x and y coprime, while g can be any positive integer.

Since $36 = 2^2 \times 3^2$, coprime x and y must each take all or none of each prime: (x, y) is $(1, 36)$, $(36, 1)$, $(4, 9)$ or $(9, 4)$. The bound 100 allows $g \leq 2$ for the first two (as $36g \leq 100$) and $g \leq 11$ for the last two (as $9g \leq 100$). The count is $2 + 2 + 11 + 11 = 26$.

Forgetting the free factor g gives 4; allowing non-coprime splits such as $(2, 18)$ adds pairs that do not work, since HCF would change. The option 25 is the number of pairs with $\text{LCM}(a, b) = 36$, a different condition.

Question 25. Answer **D**.

The routes are not equally likely, because once the robot reaches the eastern edge $x = 4$ its moves are forced. So counting routes, $\frac{\binom{6}{2}}{\binom{8}{4}} = \frac{15}{70} = \frac{3}{14}$, is wrong.

The robot passes through $(4, 2)$ exactly when it reaches the line $x = 4$ at a height of 0, 1 or 2, after which it is forced north through $(4, 2)$. That happens exactly when its fourth move east comes before

its third move north. Until one of these happens, every junction offers both choices, so this is the event that among its first six choices, made by fair coin flips, at least four are east.

The probability is

$$\frac{\binom{6}{4} + \binom{6}{5} + \binom{6}{6}}{2^6} = \frac{15 + 6 + 1}{64} = \frac{11}{32}.$$

The option $\frac{15}{64}$ counts only the routes that arrive at $(4, 2)$ from the west.

Question 26. Answer **D**.

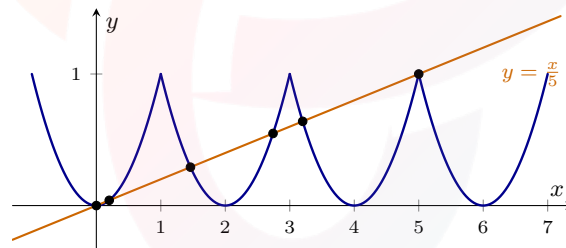
Halving an even number removes a final 0 from its binary representation, and $n \mapsto \frac{n-1}{2}$ for odd n removes a final 1; the rules say that removing a 0 leaves f unchanged and removing a 1 lowers it by 1, ending at $f(1) = 1$. So $f(n)$ is the number of 1s when n is written in base 2. (For example $6 = 110_2$ and $f(6) = 2$.)

Numbers below $128 = 2^7$ have at most seven binary digits, and those with exactly three 1s number $\binom{7}{3} = 35$. Since $100 = 1100100_2$, we must remove those between 101 and 127. Such a number starts 11 (as $1011111_2 = 95 < 100$), so it is $96 + 2^k$ with $k \leq 4$, and it exceeds 100 only for $k = 3$ or 4, giving 104 and 112.

The answer is $35 - 2 = 33$. Stopping at $\binom{7}{3} = 35$ ignores the upper limit, and the other options come from miscounting the numbers just above 100.

Question 27. Answer **B**.

The graph of f is a row of identical parabolic arches: on $2m - 1 \leq x \leq 2m + 1$ it is $y = (x - 2m)^2$, touching 0 at each even integer and rising to peaks of 1 at each odd integer. The line $y = \frac{x}{5}$ is negative for $x < 0$ and above 1 for $x > 5$, so all solutions lie in $0 \leq x \leq 5$.



On $[-1, 1]$: $x^2 = \frac{x}{5}$ gives $x = 0$ and $x = \frac{1}{5}$. On $[1, 3]$: $(x - 2)^2 = \frac{x}{5}$ gives $5x^2 - 21x + 20 = 0$, whose roots $\frac{21 \pm \sqrt{41}}{10}$ (about 1.46 and 2.74) both lie in $[1, 3]$. On $[3, 5]$: $(x - 4)^2 = \frac{x}{5}$ gives $5x^2 - 41x + 80 = 0$, that is $(5x - 16)(x - 5) = 0$, so $x = 3.2$ and $x = 5$.

At $x = 5$ the line passes through a peak of the graph, $(5, 1)$, which is shared by two arches; on both sides of it the graph is below the line, so it is a single solution and must not be counted twice. The total is 6; the answer 7 counts $x = 5$ twice, and 5 misses the solution at $x = \frac{1}{5}$, which is invisible on a rough sketch.