

HU Collection

Engineering and Science Admissions Test

Mock Set D

Mathematics 2

Time allowed: 40 minutes
Questions: 27 multiple choice
Calculators: not permitted
Written by: Kevin Hu
Date written: 28 October 2026

Name: Date sat: Score: / 27

Instructions

Answer all 27 questions. Each question has exactly one correct option and is worth one mark. No marks are deducted for wrong answers, so answer every question. Calculators are not permitted. This paper is deliberately longer than the time allows: move on from any question that is costing you more than two minutes, and come back to it at the end.

Q	1	2	3	4	5	6	7	8	9	10
Ans										

Q	11	12	13	14	15	16	17	18	19	20
Ans										

Q	21	22	23	24	25	26	27			
Ans										

1. The curve $y = \sqrt{x}$ is transformed by the following three transformations, applied in this order: first a translation of 3 units in the positive x -direction; then a stretch parallel to the x -axis with scale factor $\frac{1}{2}$ (the y -axis being invariant); then a reflection in the y -axis. The resulting curve has equation $y = g(x)$. What is the value of $g(-5)$?

- A** $\sqrt{\frac{11}{2}}$ **B** $\sqrt{7}$ **C** $g(-5)$ is not defined
D 4 **E** $\sqrt{13}$ **F** 2

2. In an experiment, two positive quantities x and y are measured. A technician plots $\log_2 y$ on the vertical axis against $\log_4 x$ on the horizontal axis, and finds that the points lie exactly on a straight line of gradient 3 which passes through the point $(1, 5)$ of this plot. For which value of x is $y = x$?

- A** $\frac{1}{2}$ **B** $\frac{1}{8}$ **C** $\frac{1}{4}$
D $\frac{1}{16}$ **E** 16 **F** $\frac{1}{64}$

3. The point $P(16, 4)$ lies on the curve $y = \sqrt{x}$. The tangent to the curve at P meets the x -axis at T , and the normal to the curve at P meets the x -axis at N . What is the area of the triangle PTN ?

- A** 128 **B** 65 **C** 66
D 64 **E** $\frac{65}{2}$ **F** 130

4. A workshop's weekly profit, in thousands of pounds, when it produces x hundred components is modelled by

$$P(x) = -x^3 + 9x^2 - 23x + 15, \quad x > 0.$$

For which values of x does the model predict a profit, that is, $P(x) > 0$?

- A** $3 < x < 5$ only **B** $0 < x < 1$ or $3 < x < 5$ **C** $1 < x < 3$ or $x > 5$
D $1 < x < 5$ **E** $1 < x < 3$ only **F** $0 < x < 1$ or $x > 5$

5. Find the sum of all the solutions of the equation

$$\tan 2x \sin 2x = \frac{3}{2}$$

in the interval $0 \leq x \leq 2\pi$, where x is in radians.

- A** 2π **B** 3π **C** 4π
D 6π **E** π **F** 8π

6. Find the complete set of real values of x for which

$$|x^3 - 4x| < 3x.$$

- A** $-\sqrt{7} < x < -1$ or $1 < x < \sqrt{7}$
B $0 < x < \sqrt{7}$
C $1 < x < \sqrt{7}$
D $1 < x < 7$
E $1 < x < \sqrt{7}$ and $x \neq 2$

F $x < -\sqrt{7}$ or $1 < x < \sqrt{7}$

7. A family of circles is defined as follows: each circle has its centre on the parabola $y = \frac{1}{4}x^2$, and each is tangent to the line $y = -1$. There is exactly one point that lies on every circle of the family. What are its coordinates?

A $(0, 0)$

B $(0, 2)$

C $(0, \frac{1}{4})$

D $(1, 0)$

E $(0, 1)$

F $(0, -1)$

8. On a large table, two straight cushions lie along the positive x -axis and the positive y -axis. A small ball is struck from the point $A(2, 3)$. It travels in straight lines, rebounds first off the cushion along the y -axis and then off the cushion along the x -axis, and then passes through the point $B(5, 1)$. At each rebound the path makes equal angles with the cushion before and after impact. What is the total distance travelled by the ball from A to B ?

A $\sqrt{65}$

B $\sqrt{53}$

C 5

D $\sqrt{13}$

E $\sqrt{85}$

F 9

9. A curve of constant width is constructed from an equilateral triangle of side 6 cm: each side of the triangle is replaced by the minor circular arc, centred at the opposite vertex, that joins the two ends of that side. What is the area, in cm^2 , of the region enclosed by the three arcs?

A $18\pi - 27\sqrt{3}$

B $18\pi + 9\sqrt{3}$

C 18π

D $36\pi - 18\sqrt{3}$

E $6\pi + 9\sqrt{3}$

F $18\pi - 18\sqrt{3}$

10. When the polynomial $p(x)$ is divided by $x^2 - 1$, the remainder is $x + 3$. What is the remainder when $(p(x))^3$ is divided by $x^2 - 1$?

A $28x + 36$

B $36x + 28$

C $27x + 36$

D $28x + 27$

E $x + 27$

F $x^3 + 9x^2 + 27x + 27$

11. A curve has gradient

$$\frac{dy}{dx} = 3x^2 - 6x$$

at every point (x, y) on it, and the x -axis is a tangent to the curve. What is the area of the finite region enclosed between the curve and the x -axis?

A 4

B $\frac{27}{8}$

C 8

D 6

E $\frac{27}{2}$

F $\frac{27}{4}$

12. A robot starts at the origin and moves 2 m due east. It then turns 90° anticlockwise and moves 1 m. It continues in this way forever: before each move it turns 90° anticlockwise, and each move is half the length of the previous move, so that the lengths of its moves form a monotonically decreasing geometric progression. The robot's position approaches a limiting point L . How far, in metres, is L from the origin?

A 4

B $2\sqrt{2}$

C $\frac{12}{5}$

D $\frac{4\sqrt{5}}{5}$

E $\sqrt{5}$

F $\frac{8}{5}$

13. A regular polygon with n sides is inscribed in a circle of radius 1. What is the smallest value of n for which the area of the polygon is greater than 3?

- A** 6 **B** 14 **C** 13
D 10 **E** 11 **F** 12

14. A calculator display shows the number 2. The calculator has only two working buttons: button S replaces the number x on the display by x^2 , and button H replaces it by $\frac{x}{2}$. What is the smallest number of button presses that will make the display show 2^{100} ?

- A** 9 **B** 8 **C** 10
D 12 **E** 7 **F** 11

15. The tangent to the curve $y = 4 - x^2$ at a point P in the first quadrant (so both coordinates of P are positive) forms a triangle with the positive coordinate axes. What is the least possible area of this triangle?

- A** $\frac{32\sqrt{3}}{9}$ **B** $\frac{64\sqrt{3}}{9}$ **C** $\frac{25}{4}$
D $\frac{16}{3}$ **E** 8 **F** $\frac{16\sqrt{3}}{9}$

16. For which values of the real constant c does the equation

$$\sqrt{x+3} + \sqrt{5-x} = c$$

have exactly two real solutions?

- A** $2\sqrt{2} \leq c < 4$ **B** $c < 4$ **C** $0 < c < 4$
D $2\sqrt{2} < c < 4$ **E** $2 \leq c < 4$ **F** $2\sqrt{2} \leq c \leq 4$

17. The region R consists of all points (x, y) in the plane satisfying

$$\sqrt{|x|} + \sqrt{|y|} \leq 2.$$

What is the area of R ?

- A** 8 **B** 32 **C** $\frac{8}{3}$
D $\frac{16}{3}$ **E** $\frac{32}{3}$ **F** $\frac{64}{3}$

18. P and Q are points on the parabola $y = x^2$ such that the tangents to the parabola at P and at Q are perpendicular to each other. What is the least possible length of the chord PQ ?

- A** $\sqrt{2}$ **B** $\frac{1}{2}$ **C** $\frac{\sqrt{2}}{2}$
D $\frac{1}{4}$ **E** 2 **F** 1

19. For a real constant k , consider the graph of

$$y = |x^4 - 8x^2 - k|.$$

For how many integer values of k does this graph have exactly three local maximum points?

- | | | |
|-------------|-------------|-------------|
| A 17 | B 30 | C 14 |
| D 31 | E 15 | F 16 |

20. The polynomial $p(x)$ has degree 3. It is known that $p(x) - 1$ is divisible by $(x - 1)^2$ and that $p(x) + 1$ is divisible by $(x + 1)^2$. What is the value of $p(3)$?

- | | | |
|---------------|-------------|--------------------------|
| A -6 | B 18 | C $-\frac{27}{2}$ |
| D -9 | E 9 | F -18 |

21. The point $A_1(1, 1)$ lies on the curve $y = x^3$. The tangent to the curve at A_1 meets the x -axis at B_1 ; the vertical line through B_1 meets the curve at A_2 ; the tangent at A_2 meets the x -axis at B_2 ; the vertical line through B_2 meets the curve at A_3 ; and so on indefinitely. For each n , let C_n be the point on the x -axis directly below A_n , and let T_n be the triangle $A_n B_n C_n$. What is the total area of all the triangles $T_1, T_2, T_3, \dots$?

- | | | |
|---------------------------|-------------------------|-------------------------|
| A $\frac{27}{130}$ | B $\frac{1}{6}$ | C $\frac{9}{38}$ |
| D $\frac{1}{2}$ | E $\frac{3}{10}$ | F $\frac{1}{4}$ |

22. An analogue clock has an hour hand of length 3 cm and a minute hand of length 5 cm, both pivoted at the centre of the face and both turning continuously at constant rates. Consider the 12-hour period that starts at noon, including the instant of noon but not the instant of midnight. On how many separate occasions during this period is the distance between the tips of the two hands a whole number of centimetres?

- | | | |
|--------------|--------------|--------------|
| A 121 | B 132 | C 154 |
| D 110 | E 144 | F 77 |

23. A machine accepts a polynomial $p(x)$ and outputs the polynomial $p(x+1) - p(x)$. The polynomial $(x^2 + x)^3$ is fed into the machine, the output is fed back into the machine, and this is repeated until the machine has been used five times in total. The final output is a linear polynomial $ax + b$. What is the value of b ?

- | | | |
|---------------|---------------|---------------|
| A 1800 | B 720 | C 2880 |
| D 360 | E 1440 | F 2160 |

24. Let $f(x) = x^3 - 21x$. For each real number $a \geq 0$, let $M(a)$ denote the greatest value taken by $f(x)$ on the interval $a \leq x \leq a + 3$. What is the least value of $M(a)$ over all $a \geq 0$?

- | | | |
|------------------------|----------------|----------------|
| A $-14\sqrt{7}$ | B -36 | C -16 |
| D -22 | E -20 | F 0 |

25. You may use the fact that $\log_{10} 2 = 0.30103$, correct to 5 decimal places. How many of the 100 numbers

$$2^1, 2^2, 2^3, \dots, 2^{100}$$

have first (leftmost) digit equal to 1?

A 33

B 25

C 31

D 30

E 10

F 29

26. A thin glass plate has two parallel faces. Whenever light meets either face, from either side, a fraction $\frac{1}{3}$ of its intensity is reflected and the remaining $\frac{2}{3}$ is transmitted; no light is absorbed, and the intensities of separate beams simply add. Two such plates are mounted parallel to each other with an air gap between them, and a beam of light strikes the first plate at right angles. The light may bounce back and forth between the four faces any number of times. What fraction of the original intensity eventually emerges beyond the second plate?

A $\frac{2}{5}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{16}{81}$

E $\frac{1}{4}$

F $\frac{9}{32}$

27. A square has vertices $(4, 4)$, $(-4, 4)$, $(-4, -4)$ and $(4, -4)$. The region R consists of all points inside the square that are closer to the centre $(0, 0)$ than to every one of the four sides of the square. What is the area of R ?

A $\frac{8(4\sqrt{2} - 5)}{3}$

B $\frac{32(4\sqrt{2} - 5)}{3}$

C $\frac{64(4\sqrt{2} - 5)}{3}$

D 16

E $\frac{32}{3}$

F $\frac{64}{3}$

End of paper