

Mathematics 1

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Instructions

Answer all 27 questions. Each question has exactly one correct option and is worth one mark. No marks are deducted for wrong answers, so answer every question. Calculators are not permitted. This paper is deliberately longer than the time allows: move on from any question that is costing you more than two minutes, and come back to it at the end.

[illegible]

1. A tank is a cuboid with a horizontal rectangular base measuring 2 m by 1.5 m, and it starts empty. Tap *A* delivers 15 litres per minute and tap *B* delivers 9 litres per minute. A drain at the bottom removes water at a constant 18 litres per minute whenever it is open and there is water in the tank.

At the start, tap *A* is turned on, but the drain has been left open by mistake. Ten minutes later tap *B* is also turned on. When the water first reaches a depth of 12 cm, the drain is closed. How long after the start does the water reach a depth of 60 cm?

- A** 5 hours 10 minutes **B** 2 hours 15 minutes **C** 2 hours 10 minutes
D 2 hours 20 minutes **E** 2 hours **F** 1 hour 15 minutes

2. The times, t minutes, taken by a group of runners to complete a course are shown in a histogram with four bars. The bar for $0 \leq t < 10$ is 1 cm high, the bar for $10 \leq t < 15$ is 3 cm high, the bar for $15 \leq t < 20$ is 4 cm high and the bar for $20 \leq t < 40$ is 0.75 cm high. Exactly 30 runners had times in the interval $10 \leq t < 15$.

Using linear interpolation, and taking the lower and upper quartiles to be the $\frac{n}{4}$ th and $\frac{3n}{4}$ th values, where n is the number of runners, estimate the interquartile range in minutes.

- A** $\frac{15}{2}$ **B** 9 **C** $\frac{20}{3}$
D $\frac{35}{3}$ **E** 10 **F** $\frac{25}{3}$

3. For how many real values of k does the equation

$$(k-1)x^2 - 4x + (k+2) = 0$$

have exactly one real solution for x ?

- A** 5 **B** 4 **C** 1
D 3 **E** 2 **F** 6

4. Find the sum of all real solutions of the equation

$$(x^2 - 3x)^2 + (x^2 - 3x) - 6 = 0.$$

- A** -3 **B** 2 **C** 6
D 3 **E** 0 **F** -1

5. Four people are chosen at random. Assume that each person is equally likely to have been born in any of the 12 months, independently of the others. What is the probability that their four birth months include exactly three different months?

- A** $\frac{55}{864}$ **B** $\frac{1}{4}$ **C** $\frac{55}{144}$
D $\frac{55}{72}$ **E** $\frac{55}{216}$ **F** $\frac{41}{96}$

6. The numbers $x = 12$ and $y = 4$ are each given correct to the nearest integer. What is the upper bound of

$$\frac{x+y}{x-y}?$$

- | | | |
|-------------------------|-------------------------|-------------------------|
| A $\frac{16}{7}$ | B $\frac{15}{7}$ | C $\frac{17}{8}$ |
| D $\frac{16}{9}$ | E $\frac{17}{7}$ | F 2 |

7. How many of the following eight numbers are factors of $2^{20} - 1$?

3, 5, 7, 11, 13, 25, 31, 41

- | | | |
|------------|------------|------------|
| A 3 | B 6 | C 5 |
| D 8 | E 7 | F 4 |

8. Priya cycles from P to Q and Quentin cycles from Q to P along the same road. They set off at the same time and each rides at a constant speed. They pass each other after 2 hours, and Priya reaches Q exactly 1 hour 20 minutes after they pass. How long does Quentin's whole journey take?

- | | | |
|------------------|-----------------------------|-----------------------------|
| A 6 hours | B 4 hours 40 minutes | C 3 hours 20 minutes |
| D 3 hours | E 4 hours | F 5 hours |

9. AB is a diameter of a circle, and the tangent to the circle at B is drawn. A straight line through A meets the circle again at C and meets the tangent at T , with C between A and T . Given that $AC = 9$ and $CT = 16$, find the distance from C to the line AB .

- | | | |
|-------------------------|-------------------------|-------------------------|
| A 12 | B $\frac{48}{5}$ | C $\frac{36}{5}$ |
| D $\frac{15}{2}$ | E $\frac{27}{5}$ | F 9 |

10. A regular hexagon has area 60 cm^2 . Label its sides 1 to 6 in order around the hexagon. The midpoints of sides 1, 3 and 5 are joined to form a triangle. What is the area of this triangle in cm^2 ?

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|-------------|---------------|-------------|
| A 15 | B 22.5 | C 20 |
| D 24 | E 30 | F 25 |

11. What is the smallest positive integer n such that $12n$ is a perfect square and $18n$ is a perfect cube?

- | | | |
|-------------|--------------|-------------|
| A 36 | B 144 | C 24 |
| D 72 | E 12 | F 6 |

12. Evaluate

$$\frac{1}{2\sqrt{1} + 1\sqrt{2}} + \frac{1}{3\sqrt{2} + 2\sqrt{3}} + \frac{1}{4\sqrt{3} + 3\sqrt{4}} + \cdots + \frac{1}{100\sqrt{99} + 99\sqrt{100}}.$$

- | | | |
|--------------------------|---------------------------|--------------------------|
| A $\frac{11}{10}$ | B 1 | C $\frac{10}{11}$ |
| D 9 | E $\frac{99}{100}$ | F $\frac{9}{10}$ |

13. The positive real number x satisfies $x - \frac{1}{x} = 3$. Find the value of

$$x^6 - \frac{1}{x^6}.$$

- A 396 B $120\sqrt{13}$ C $36\sqrt{13}$
 D $360\sqrt{13}$ E $-360\sqrt{13}$ F 1296

14. A triangle has sides of length 5, 7 and 8. What is the radius of the circle inscribed in the triangle?

- A $\frac{\sqrt{3}}{2}$ B $\sqrt{3}$ C $\frac{5\sqrt{3}}{4}$
 D 2 E $\frac{7\sqrt{3}}{3}$ F $2\sqrt{3}$

15. The five numbers 3, 7, 12, 15 and x , where x is a real number, have a mean equal to their median. What is the sum of all possible values of x ?

- A 21 B 30 C 23
 D $\frac{37}{4}$ E $\frac{121}{4}$ F $\frac{129}{4}$

16. A solid wooden cube has edges of length 10 cm. Three square tunnels, each with cross-section 4 cm by 4 cm, are cut right through the cube. Each tunnel runs between the centres of a pair of opposite faces, with its sides parallel to the edges of the cube. What is the total surface area, inside and outside, of the resulting solid, in cm^2 ?

- A 888 B 984 C 1080
 D 600 E 696 F 792

17. A bag contains 4 red and 3 blue counters. Counters are taken out one at a time, at random and without replacement, until the bag is empty. What is the probability that the second blue counter is taken out before the third red counter?

- A $\frac{22}{35}$ B $\frac{3}{7}$ C $\frac{1}{2}$
 D $\frac{13}{35}$ E $\frac{4}{7}$ F $\frac{18}{35}$

18. Three different vertices of a cube are chosen at random, with every set of three vertices equally likely. What is the probability that they form a right-angled triangle?

- A $\frac{3}{7}$ B $\frac{5}{7}$ C $\frac{4}{7}$
 D $\frac{1}{7}$ E $\frac{1}{2}$ F $\frac{6}{7}$

19. Two chords AC and BD of a circle are perpendicular and meet at a point X inside the circle. Given that $AX = 2$, $XC = 6$ and $BX = 3$, find the radius of the circle.

- A $2\sqrt{5}$ B $\frac{\sqrt{65}}{2}$ C 4
 D $\frac{7}{2}$ E $\sqrt{65}$ F $\frac{\sqrt{61}}{2}$

20. A sealed hollow cone of height 12 cm contains some water. When the cone stands on its circular base, with its vertex pointing up, the water is 6 cm deep. The cone is turned over so that it balances with its vertex pointing down and its base horizontal. How deep, in cm, is the water now? (Ignore the thickness of the walls.)

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|-------------------------|-------------------------|---------------|
| A $3\sqrt[3]{7}$ | B $6\sqrt[3]{7}$ | C 6 |
| D $3\sqrt{14}$ | E $6\sqrt[3]{4}$ | F 10.5 |

21. An escalator moves downwards at a constant speed. Ria walks down it, from top to bottom, while it is moving. When she walks at 1 step per second she takes 30 steps to reach the bottom. When she walks at 3 steps per second she takes 45 steps. How many steps would she take to walk down the escalator if it were switched off?

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|-------------|-------------|-------------|
| A 45 | B 90 | C 60 |
| D 75 | E 50 | F 40 |

22. How many ordered pairs (x, y) of integers satisfy

$$xy = 4x + 6y?$$

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|-------------|-------------|-------------|
| A 4 | B 8 | C 12 |
| D 15 | E 24 | F 16 |

23. Asha and Bilal take turns to roll a fair six-sided die, with Asha going first. Asha wins if she rolls a 6 on her turn; Bilal wins if he rolls a 5 or a 6 on his turn. They keep going until someone wins. What is the probability that Asha wins?

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|-------------------------|-------------------------|------------------------|
| A $\frac{1}{3}$ | B $\frac{5}{8}$ | C $\frac{3}{8}$ |
| D $\frac{6}{11}$ | E $\frac{3}{11}$ | F $\frac{1}{2}$ |

24. Triangle ABC has $AB = 13$, $BC = 14$ and $CA = 15$. The bisector of angle BAC meets BC at E , and M is the midpoint of BC . What is the area of triangle AEM ?

- | | | |
|-------------|------------------------|------------------------|
| A 3 | B 6 | C $\frac{3}{2}$ |
| D 12 | E $\frac{9}{2}$ | F 9 |

25. $ABCD$ is a square of side 12, labelled in order. E is the midpoint of BC and F is the midpoint of CD . The line segments AE and BF meet at P . What is the area of the quadrilateral $APFD$?

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|--------------------------|--------------------------|--------------------------|
| A 76 | B 72 | C 84 |
| D $\frac{384}{5}$ | E $\frac{432}{5}$ | F $\frac{396}{5}$ |

26. A right-angled triangle has hypotenuse of length 10, and the circle inscribed in it has radius 1. What is the area of the triangle?

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|-------------|-------------------------|-------------|
| A 11 | B $\frac{21}{2}$ | C 22 |
| D 10 | E 12 | F 24 |

27. Two circles, of radii 4 and 9, touch each other externally, and both touch a straight line ℓ , on the same side of it. A third circle, smaller than both, fits in the gap enclosed by the two circles and ℓ : it touches ℓ and touches each of the other two circles externally. What is the radius of the third circle?

A $\frac{36}{13}$
D $\frac{36}{25}$

B 1
E $\frac{3}{2}$

C $\frac{13}{9}$
F $\frac{6}{5}$



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